Student ID: ______________________ Name: ________________________

MA205B - Differential Equations I - Quiz 1

TUESDAY, OCTOBER 2, 2012

1. Solve the following **THREE** differential equations.

$$\frac{dy}{dx} = \frac{y}{x} + 2x + 1,$$

Linear

$$\frac{dy}{dx} - \frac{y}{x} = 2x+1$$

Let $\mu(x) = e^{-\int \frac{1}{x}dx} = e^{-\ln x} = e^{\ln \frac{1}{x}}$

$= \frac{1}{x}$

$\cancel{\frac{d}{dx}\left(\frac{1}{x}y\right) =}$

$$\frac{d}{dx}\left(\mu(x)\, y\right) = \mu(x)\left(2x+1\right)$$

$$\frac{d}{dx}\left(\frac{1}{x}y\right) = 2 + \frac{1}{x}$$

$$\frac{1}{x}y = 2x + \ln x + C$$

$$y = 2x^2 + x\ln x + Cx$$

$$dy - t^2\,(1+y)\,dt = 0,$$

Separable

$$\frac{dy}{dt} = t^2(1+y)$$

$$\frac{1}{1+y}\frac{dy}{dt} = t^2$$

$$\int \frac{1}{1+y}\,dy = \int t^2\,dt$$

$$\ln|1+y| = \frac{1}{3}t^3 + C_1$$

$$|1+y| = e^{\frac{1}{3}t^3 + C_1}$$

$$1+y = Ce^{\frac{1}{3}t^3}, \quad C = \pm e^{C_1}$$

$$y = Ce^{\frac{1}{3}t^3} - 1$$

$$\underbrace{\cos\theta}_{=M}\, dr + \underbrace{(e^{\theta} - r\sin\theta)}_{=N}\, d\theta = 0$$

Test for Exactness: $\left(\frac{\partial M}{\partial \theta} = \frac{\partial N}{\partial r}\right)$

$$\left.\begin{aligned}\frac{\partial M}{\partial \theta} &= -\sin\theta \\ \frac{\partial N}{\partial r} &= -\sin\theta\end{aligned}\right\}$$ These are equal. Thus, the equation is exact.

Look for $F(r,\theta)$ such that $\frac{\partial F}{\partial r} = \cos\theta = M$ and $\frac{\partial F}{\partial \theta} = \underbrace{e^{\theta} - r\sin\theta}_{=N}$.

$$\frac{\partial F}{\partial r} = \cos\theta \implies F = r\cos\theta + g(\theta)$$

$$\implies \frac{\partial F}{\partial \theta} = -r\sin\theta + g'(\theta)$$

Compare this to N.

$$e^{\theta} - r\sin\theta = \frac{\partial F}{\partial \theta} = -r\sin\theta + g'(\theta)$$

So $$e^{\theta} = g'(\theta)$$

$$e^{\theta} - c = g(\theta)$$

So $$F(r,\theta) = r\cos\theta + e^{\theta} - c$$

So, the solution is $F(r,\theta) = 0$

$$r\cos\theta + e^{\theta} = c$$