1. Enter "I" in the bubble sheet as your answer to Question 1 now.

2. Evaluate $\int_0^{\pi/8} \sec^2(2t)\,dt$.

(A) -2.00	(B) -1.0	(C) -0.6	(D) -0.3	(E) 0.00
(F) 0.3	(G) 0.6	(H) 1.0	(I) 2.0	(J) 3.0

$$\int_0^{\pi/8} \sec^2(2t)\,dt = \tfrac{1}{2}\tan(2t)\Big|_0^{\pi/8}$$

$$= \tfrac{1}{2}\left[\tan(\pi/4) - \tan(0)\right]$$

$$= \tfrac{1}{2}(1-0)$$

$$= \tfrac{1}{2}$$

3. The rate of water flow into an initially empty container is $(50 - 2t)$ liters per minute at time t (in minutes). How much water flows into the container during the interval from $t = 10$ to $t = 20$ minutes?

(A) 200	(B) 400	(C) 600	(D) 800	(E) 1,000
(F) 1,200	(G) 1,400	(H) 1,600	(I) 1,800	(J) 2,000

$$\frac{dV}{dt} = 50 - 2t$$

$$\therefore V = \int_{10}^{20} (50-2t)\,dt$$

$$= \left[50t - t^2\right]_{10}^{20}$$

$$= (50(20) - 20^2) - (50(10) - 10^2)$$

$$= 1000 - 400 - 500 + 100$$

$$= 200$$

4. Determine the value of the derivative of $y = 6^{x\sqrt{x}}$ at the point $x = 1$.

~~(A) 3~~	(B) 6	(C) 9	(D) 12	(E) 14
(F) 16	(G) 18	(H) 20	(I) 22	(J) Does not exist

$$y = 6^{x\sqrt{x}} = 6^{x^{3/2}}$$

$$\ln y = x^{3/2} \ln(6)$$

$$\frac{1}{y}\frac{dy}{dx} = \tfrac{3}{2} x^{1/2} \ln(6)$$

$$\frac{dy}{dx} = \tfrac{3}{2} \ln(6)\, x^{1/2} \cdot 6^{x^{3/2}}$$

At $x=1$, $\frac{dy}{dx} = \tfrac{3}{2}\ln(6)\cdot 1 \cdot 6^{1} = 9\ln(6) \approx 16.1258$

5. Calculate $\int_0^{1/3} \frac{1}{1+9x^2}\,dx$.

(A) 0.10	(B) 0.15	(C) 0.20	(D) 0.25	(E) 0.30
(F) 0.40	(G) 0.50	(H) 0.60	(I) 0.80	(J) 1.00

$$\int_0^{1/3} \frac{1}{1+9x^2}\,dx = \int_0^{1/3} \frac{1}{1+(3x)^2}\,dx$$

$$= \tfrac{1}{3}\tan^{-1}(3x)\Big|_0^{1/3}$$

$$= \tfrac{1}{3}\left(\tan^{-1}(1) - \tan^{-1}(0)\right)$$

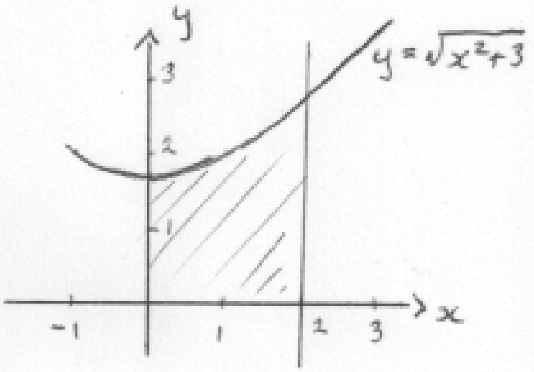
$$= \tfrac{1}{3}\left(\pi/4 - 0\right)$$

$$= \pi/12$$

$$\approx 0.2618$$

6. Calculate the volume of the solid formed by rotating the region bounded by the curves $y = \sqrt{x^2+3}$, $x = 0$ and $x = 2$ about the x-axis.

(A) 5 (B) 15 (C) 25 (D) 35 (E) 45
(F) 55 (G) 65 (H) 75 (I) 85 (J) 95



$$V = \int_0^2 \pi r^2 \, dx$$

$$= \pi \int_0^2 \left(\sqrt{x^2+3}\right)^2 dx$$

$$= \pi \int_0^2 (x^2+3)\, dx$$

$$= \pi \left[\tfrac{1}{3} x^3 + 3x \right]_0^2$$

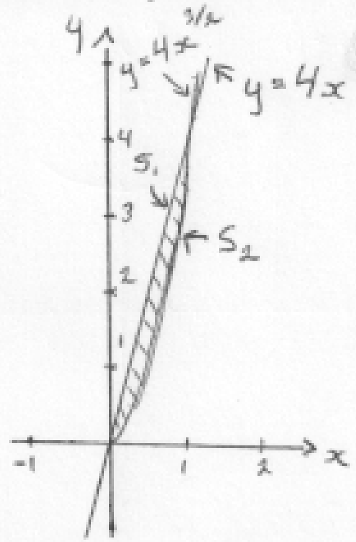
$$= \pi \left[\tfrac{1}{3}(2^3) + 3(2) - (\tfrac{1}{3}(0) + 0) \right]$$

$$= \pi \left(\tfrac{8}{3} + 6 \right)$$

$$= \frac{26\pi}{3}$$

$$\approx 27.2271$$

7. (5 points) Find the minimum length of fence needed to enclose a field bounded by the curves $y = 4x$ and $y = 4x^{3/2}$.



$$y = 4x$$
$$y' = 4$$
$$(y')^2 = 16$$

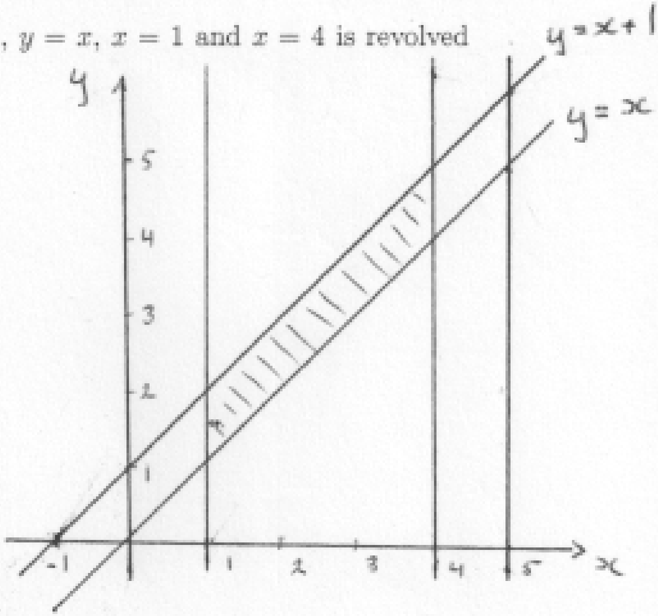
$$y = 4x^{3/2}$$
$$y' = 6x^{1/2}$$
$$(y')^2 = 36x$$

$$4x^{3/2} = 4x$$
$$x^{3/2} - x = 0$$
$$x(x^{1/2} - 1) = 0$$
$$x = 0 \text{ or } x = 1$$

$$S = S_1 + S_2$$
$$= \int_0^1 \sqrt{1+16}\,dx + \int_0^1 \sqrt{1+34x}\,dx$$
$$= \int_0^1 \sqrt{17}\,dx + \int_0^1 \sqrt{1+36x}\,dx$$
$$= \sqrt{17}\,x \Big|_0^1 + \frac{2}{3}(1+36x)^{3/2} \cdot \frac{1}{36} \Big|_0^1$$
$$= \sqrt{17} + \frac{1}{54}\left[(1+36)^{3/2} - (1+0)^{3/2}\right]$$
$$= \sqrt{17} + \frac{1}{54}(37^{3/2} - 1)$$
$$\approx 8.2724$$

8. (5 points) The region bounded by the curves $y = x + 1$, $y = x$, $x = 1$ and $x = 4$ is revolved around the line $x = 5$.

Find the volume of the resulting solid.



By cylindrical shells:

$$V = \int_1^4 2\pi r h \, dx$$

$$= 2\pi \int_1^4 (5-x)(x+1-x)\, dx$$

$$= 2\pi \int_1^4 (5-x)\, dx$$

$$= 2\pi \left[5x - \tfrac{1}{2}x^2\right]_1^4$$

$$= 2\pi \left[5(4) - \tfrac{1}{2}(4^2) - (5(1) - \tfrac{1}{2}(1^2))\right]$$

$$= 2\pi \left(20 - 8 - 5 + \tfrac{1}{2}\right)$$

$$= 2\pi \left(7\tfrac{1}{2}\right)$$

$$= 15\pi$$

The volume of the solid is 15π.

9. (5 points) Compute:

[1 point] (a) value of $\sec^{-1}(3)$

$$\sec^{-1}(3) = \cos^{-1}(1/3) \approx 1.2310$$

[2 points] (b) derivative of $g(x) = \cosh(5x)$ using definition of the hyperbolic cosine, and express the results in terms of a hyperbolic function;

$$g(x) = \cosh(5x) = \tfrac{1}{2}(e^{5x} + e^{-5x})$$

$$g'(x) = \tfrac{1}{2}(5e^{5x} - 5e^{-5x}) = 5 \cdot \tfrac{1}{2}(e^{5x} - e^{-5x}) = 5\sinh(5x)$$

[2 points] (c) derivative of $f(x) = \sinh^{-1}(2x)$ using properties of the inverse functions.

Let $u = \sinh^{-1}(2x)$

Then $\sinh(u) = \sinh(\sinh^{-1}(2x)) = 2x$ ①

$$\frac{d}{dx}\sinh(u) = 2$$

But $\frac{d}{dx}\sinh(u) = \cosh(u)\frac{du}{dx}$

So $\cosh(u)\frac{du}{dx} = 2$

$$\frac{du}{dx} = \frac{2}{\cosh(u)}$$

$\cosh^2(u) - \sinh^2(u) = 1$

$\cosh(u) = \sqrt{1 + \sinh^2(u)}$

$$= \frac{2}{\sqrt{1+\sinh^2(u)}}$$

Substitute in ①

$$= \frac{2}{\sqrt{1+(2x)^2}}$$

$$= \frac{2}{\sqrt{1+4x^2}}$$