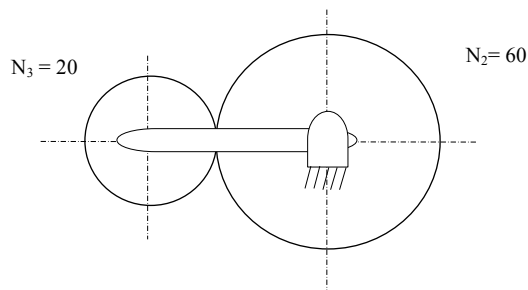


## FINAL EXAMINATION

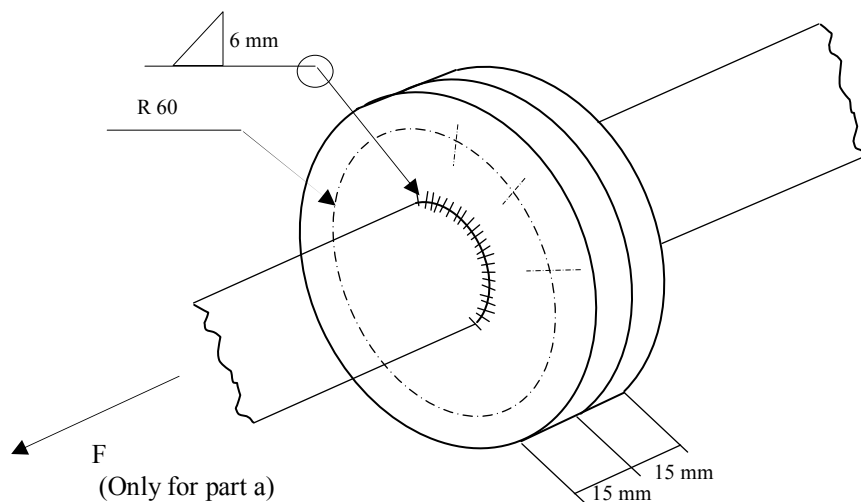
Open book. Calculators permitted. Attempt all 5 questions. Time: 3hrs.

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- 1- A Deep-Groove ball bearing is under an axial load of 1400 N and radial load of 4500 N. It is required to work  $58.5 \times 10^6$  cycles of inner ring rotation with 99% reliability. Select the proper bearing from table 11-2 of the text book. (15 points)
  
- 2- A compression helical spring has mean coil diameter of  $D = 0.5625$  inch and 21 active coils. 28.21 lbf is required to push the spring from its free length of  $L_0 = 4$  inches to a length of 3 inches. What is the safety factor in the spring when it is compressed to its solid length. The spring has plain ends and is made from music wire. Assume the shear modulus of elasticity  $G = 11.8 \times 10^6$  psi and shear yield strength is found from  $S_{sy} = 0.56 S_{ut}$ . (20 points)
  
- 3- In an amusement park a planetary gear system is used. The arm rotates 100 rpm ccw. An electric motor which is installed on the arm rotates the gear 3 at 50 rpm cw with respect to the arm. Find the speed and direction of rotation of the sun gear 2. Gear 3 has 20 teeth and sun gear 2 has 60 teeth. (15 points)



- 4- A full journal bearing has a journal with a diameter of 30 mm. The bushing bore has a diameter of 30.05 mm. The  $l/d$  (length to diameter ratio) is equal to one. The bushing load is 2 kN, and the journal rotates at 1100 rev/min. The average viscosity is 52.4 mPa.s ( $52.4 \times 10^{-3}$  Pa.s). Find the minimum film thickness, the power loss, the side flow and temperature rise in degrees Celsius. (15 points)
- 5- Two shafts are connected using two flanges. The shafts and flanges are made from AISI No. 1020 CD steel. The shafts are welded to the flanges using E70xx electrodes with 6 mm fillet. The flanges are attached together using 10 of M10 $\times$ 1.5 bolts with property class of 8.8. The welds are on an average diameter of 50 mm and the bolts are on diameter of 120 mm. Do not consider any stress concentration. All materials fail based on Von-Mises theory. The threads in bolts stop right after the nuts leaving the grip length of the bolts unthreaded.
- a) The tensile force  $F$  in the shafts varies between 50000 N and 80000 N and there is no other load applied. Use Goodman criteria and for unlimited life:
- determine the safety factor in the bolts if the preload in each bolt is 10000 N. Assume endurance limit in the bolts is  $S_e = 200$  MPa
  - determine the safety factor in the welds. Consider  $S_{su} = 0.59 S_{ut}$ .
- b) There is no tensile force applied but the shafts are used to transfer 12000 watts of power at 100 rpm.
- determine the safety factor in the bolts when there is no preload in them.
  - determine the safety factor in the welds. (35 points)



### Answer 1

$$L_{10} = 58.5 \times 10^6 / 0.21 = 278.5 \times 10^6 \text{ rev}$$

1<sup>st</sup> iteration)

$$d = 20 \rightarrow C_{10} = 12.7 \text{ kN}, C_0 = 6.2 \text{ kN}$$

$$F_a/C_0 = 1400/6200 = 0.226 \rightarrow e = 0.36, F_a/V_{Fr} = 1400/4500 = 0.31 < e \rightarrow X_1 = 1,$$

$$Y_1 = 0 \text{ and } F_e = 1 \times 4500 + 0 = 4500 \text{ N}$$

$$12.7 \times (10^6)^{1/3} = 4500 \times L_{10}^{1/3} \rightarrow L_{10} = 22.5 \times 10^6 < 278.5 \times 10^6 \text{ needs stronger bearing.}$$

2nd iteration)

$$d = 30 \rightarrow C_{10} = 19.5 \text{ kN}, C_0 = 10 \text{ kN}$$

$$F_a/C_0 = 1400/10000 = 0.14 \rightarrow e = 0.32, F_a/V_{Fr} = 1400/4500 = 0.31 < e \rightarrow X_1 = 1,$$

$$Y_1 = 0 \text{ and } F_e = 1 \times 4500 + 0 = 4500 \text{ N}$$

$$19.5 \times (10^6)^{1/3} = 4500 \times L_{10}^{1/3} \rightarrow L_{10} = 81.4 \times 10^6 < 278.5 \times 10^6 \text{ needs stronger bearing.}$$

3rd iteration)

$$d = 40 \rightarrow C_{10} = 30.7 \text{ kN}, C_0 = 16.6 \text{ kN}$$

$$F_a/C_0 = 1400/16600 = 0.084 \rightarrow e = 0.28, F_a/V_{Fr} = 1400/4500 = 0.31 > e \rightarrow X_2 =$$

$$0.56, Y_2 = 1.55 \text{ and } F_e = 0.56 \times 4500 + 1.55 \times 1400 = 4690 \text{ N}$$

$$30.7 \times (10^6)^{1/3} = 4690 \times L_{10}^{1/3} \rightarrow L_{10} = 280.48 \times 10^6 > 278.5 \times 10^6 \text{ just above the required life and is acceptable.}$$

Note that, in each iteration, instead of checking the life it is possible to check  $C_{10}$  against the value in the table. for example in iteration 3 if we check for  $C_{10}$  we have:

$$C_{10} \times (10^6)^{1/3} = 4690 \times (278.5 \times 10^6)^{1/3} \rightarrow C_{10} = 30.63 \approx 30.7 \text{ acceptable}$$

### Answer 2

$$k = F / \delta = d^4 G / 8D^3 N \rightarrow 28.21 / (4-3) = d^4 \times 11.8 \times 10^6 / (8 \times 0.5625^3 \times 21) \rightarrow d = 0.092 \text{ in}$$

$$L_s = (N_t + 1) d = (21+1) \times 0.092 = 2.024 \text{ in}$$

$$\delta = 4 - 2.024 = 1.976 \text{ in}$$

$$F = K \delta = 28.21 \times 1.976 = 55.74 \text{ lbf}$$

$$C = D/d = 0.5625/0.092 = 6.11$$

$$K_B = (4C+2)/(4C-3) = 1.233$$

$$S_{ut} = A/d^m = 201 / 0.092^{0.145} = 284.1 \text{ kpsi}$$

$$\tau = K_B \times 8FD / (\pi d^3) = 126426 \text{ psi}$$

$$S_{sy} = 0.56 \times 284.1 = 159.1, n = S_{sy} / \tau = 159.1/126.4 = 1.26$$

### Answer 3

First gear is number 2 and last gear is number 3. cw is positive and ccw is negative

$$e = -N_2/N_3 = -60/20 = -3$$

$$e = (n_L - n_A) / (n_f - n_A) = 50 / (n_f - (-100)) = -3$$

$$n_f = -116.67 \text{ rpm. So the sun rotates 116.67 rpm ccw}$$

Note that when it is told gear 3 rotates 50 rpm with respect to arm it means that

$$n_3 - n_A = n_{3/A} = 50$$

Answer 4

$$d = 30 \text{ mm and } c = (30.05 - 30) / 2 = 0.025 \text{ mm}$$

$$L/d = 1 \text{ then } L = 30 \text{ mm}$$

$$W = 2 \text{ kN} = 2000 \text{ N,}$$

$$N = 1100 \text{ rev/min} = 18.33 \text{ rev/s}$$

$$P = W/Ld = 2000 / (30 \times 10^{-3} \times 30 \times 10^{-3}) = 2.22 \times 10^6 \text{ Pa}$$

$$S = (r/c)^2 \mu N / P = (15/0.025)^2 \times 52.4 \times 10^6 \times 18.33 / 2.22 \times 10^6 = 0.156$$

$$\text{From figure 12-16} \rightarrow h_o/c = 0.45 \rightarrow h_o = 0.01125 \text{ mm}$$

$$\text{From figure 12-18} \rightarrow f_t/c = 3.8 \rightarrow f = 6.33 \times 10^{-3}$$

$$p = T\omega = (fWr)(2\pi N) = 6.33 \times 10^{-3} \times 2000 \times 15 \times 10^{-3} \times 2 \times \pi \times 18.33 = 21.9 \text{ watts}$$

$$\text{From figure 12-19} \rightarrow Q/rcNL = 4.25 \text{ and}$$

$$\text{From figure 12-20} \rightarrow Q_s/Q = 0.65 \rightarrow Q_s = 5.7 \times 10^{-7} \text{ m}^3/\text{s}$$

$$\text{From figure 12-24} \rightarrow 0.12\Delta T/P = 1.3 \rightarrow \Delta T = 24.05 \text{ }^\circ\text{C}$$

Answer 5

Bolts: Property class 8.8 from table 8-11:  $S_p = 600$ ,  $UTS = 830$ ,  $S_y = 660 \text{ MPa}$

Weld: E70xx from table 9-3:  $UTS = 482$ ,  $S_y = 393 \text{ MPa}$

Shaft and flange: AISI No. 1020 CD from table A-20:  $UTS = 470$ ,  $S_y = 390 \text{ MPa}$

$$\text{Weld area: } A = 0.707 \times \pi \times d \times h = 0.707 \times \pi \times 50 \times 6 = 666.33 \text{ mm}^2$$

$$\text{Bolt area: from table 8-1} \rightarrow A_t = 58 \text{ mm}^2, A_d = \pi \times d^2 / 4 = \pi \times 10^2 / 4 = 78.54 \text{ mm}^2$$

a)i)

$$K_{\text{bolt}} = AE/L = 78.54 \times 10^{-6} \times 207 \times 10^9 / 30 \times 10^{-3} = 542 \times 10^6 \text{ N/m}$$

To calculate  $K_m$  for member there are two methods available:

$$\text{I) } K_m / (Ed) = A \exp(Bd/l) \rightarrow$$

$$K_m / (207 \times 10^9 \times 0.01) = 0.78715 \times \exp(0.62783 \times 0.01/0.03)$$

$$K_m = 2 \times 10^9 \text{ N/m, OR}$$

$$\text{II) } k_m = (0.5774 \pi Ed) / [2 \ln(5(0.5774l + 0.5d) / (0.5774l + 2.5d))]$$

By substituting the values in this equation  $K_m = 1.946 \times 10^9 \text{ N/m}$

$$C = K_b / (K_b + K_m) = 0.218$$

$$F_{\text{bolt}}^{\text{min}} = CF^{\text{min}} + \text{preload} = 0.218 \times (50000/10) + 10000 = 11090 \text{ N}$$

$$\text{Then } \sigma_{\text{bolt}}^{\text{min}} = F_{\text{bolt}}^{\text{min}} / A_t = 11090 / 58 = 191.2 \text{ MPa}$$

$$F_{\text{bolt}}^{\text{max}} = CF^{\text{max}} + \text{preload} = 0.218 \times (80000/10) + 10000 = 11744 \text{ N}$$

$$\text{Then } \sigma_{\text{bolt}}^{\text{max}} = F_{\text{bolt}}^{\text{max}} / A_t = 11744 / 58 = 202.5 \text{ MPa}$$

$$\sigma_{\text{mid}} = (\sigma_{\text{bolt}}^{\text{max}} + \sigma_{\text{bolt}}^{\text{min}}) / 2 = 196.9 \text{ MPa, } \sigma_{\text{amp}} = (\sigma_{\text{bolt}}^{\text{max}} - \sigma_{\text{bolt}}^{\text{min}}) / 2 = 5.65 \text{ MPa}$$

$$\text{Goodman: } n \sigma_{\text{amp}} / S_e + n \sigma_{\text{mid}} / S_{ut} = 1 \rightarrow n 5.65 / 200 + n 196.9 / 830 = 1 \rightarrow n = 3.77$$

a)ii)

$$\tau_{\text{max}} = 80000 / 666.33 = 120 \text{ MPa, } \tau_{\text{min}} = 50000 / 666.33 = 75 \text{ MPa}$$

$$\tau_{\text{amp}} = (\tau_{\text{max}} - \tau_{\text{min}}) / 2 = 22.5 \text{ MPa, } \tau_{\text{mid}} = (\tau_{\text{max}} + \tau_{\text{min}}) / 2 = 97.5 \text{ MPa}$$

$$S'_e = 470 \times 0.5 = 235 \text{ MPa}$$

$$K_a = a S_{ut}^b = 272 \times 470^{-0.995} = 0.597$$

$$K_b = 1, K_c = 0.59$$

$$S_e = 235 \times 0.597 \times 0.59 = 82.77 \text{ MPa, } S_{su} = 470 \times 0.59 = 277.3$$

$$\text{Goodman: } n \tau_{\text{amp}} / S_e + n \tau_{\text{mid}} / S_{su} = 1 \rightarrow n 22.5 / 82.77 + n 97.5 / 277.3 = 1 \rightarrow n = 1.6$$

b)i)

$$100 \text{ rpm} = 10.47 \text{ rad/s, } T = P/\omega = 12000 / 10.47 = 1146 \text{ Nm}$$

$$F = T/r = 1146/0.06 = 19100 \text{ N, in each bolt } F1 = 19100/10 = 1910 \text{ N}$$

$$\tau = F1/A = 1910/78.5 = 24.33 \text{ MPa, } n = 0.577 \text{ Sy} / \tau = 0.577 \times 660 / 24.33 = 15.67$$

b)ii)

$$\text{for weld } J = Ar^2 = 666.33 \times 25^2 = 416.46 \times 10^3 \text{ mm}^4$$

$$\text{OR from table 9-1 } J_u = 2\pi r^3 \text{ and } J = 2\pi r^3 \times 6 \times 0.707 = 416.46 \times 10^3 \text{ mm}^4$$

$$\tau = Tc/J = 1146 \times 10^3 \times 25 / 416.46 \times 10^3 = 68.8 \text{ MPa}$$

$$n = 0.577 \text{ Sy} / \tau = 0.577 \times 390 / 68.8 = 3.27$$