MATH 1004A Midterm

July 6, 2014

6:35 - 7:55

Full Name: ______________________________

Student #: ______________________________

- Each question is worth 10 marks. There are 5 questions.
- Non-programmable calculators are allowed, but not required.
- Be calm and don't rush. You have plenty of time for this test.

Good luck!

Question 1.

a. Find the following **limits**:

(a) $\lim_{x \to 0} x^5 - e^x$

(b) $\lim_{x \to 3} \dfrac{x^2 - 4x + 3}{x - 3}$

(c) $\lim_{x \to 0} \dfrac{x}{\sin 3x}$

b. State the conditions for a function $f(x)$ to be **continuous** at the point $x = a$.

c. We know by the concept of dominance that $\lim_{x \to +\infty} \frac{x^2}{e^x} = 0$. Use **L'Hôpital's Rule** to prove it.

Question 2.

a. Differentiate the function $f(x) = x^2 - 3$ by **first principles**.

b. **Differentiate** the following functions:

(a) $f(x) = e^{\sin x}$

(b) $f(x) = x^{177} + 177^x$

(c) $f(x) = (\log_6 x)(\tan^{-1} x)$

c. Use **implicit differentiation** to find y' for the following relation:

$$x^2 y - \frac{y}{x} = \ln y + e^x$$

Question 3.

a. State the **Intermediate Value Theorem**.

b. **Approximate** the value of $\sqrt{27}$, using the derivative of $f(x) = \sqrt{x}$ and the fact that $\sqrt{25} = 5$.

c. Suppose we want to find $\sqrt{27}$ by **Newton's method** instead. Let $f(x) = x^2 - 27$ and let our initial guess be $x_0 = 5$. Find x_1 and x_2, and explain why the xs will approach $\sqrt{27}$.

Question 4.

a. A circular puddle of oil is growing at a constant rate of 15 cm^2/s. At the moment when the area of the puddle is 200 cm^2, how fast is the puddle's radius growing? (If you have a calculator, round your answer to two decimal places; if not, leave it in exact form.)

b. Prove that there is no tangent to the graph of $y = e^x$ that passes through the point (0,2). (Hint: Write the equation of a general tangent first.)

Question 5.

Sketch the graph of the function $f(x) = \dfrac{x+1}{x-1}$.