Solutions

MAT2377 3X: Probability and Statistics for Engineers
Instructor: Aziz Khanchi

Midterm Test
Spring 2014

Surname ____________________ First Name ____________________

Student # ____________________

Take your time to read the entire paper before you begin to write, and read each question carefully. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- There are two short answer questions and 9 multiple choice questions. Detail of your work is required for short answer questions.
- Do **not** detach the exam booklet. You have to return the complete stapled booklet.
- You have 80 minutes to complete this exam.
- Only answers of MC questions recorded in the table on the second page will be marked.
- This is an open book exam, and notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted.**
- Where it is possible to check your work, do so.

*Good Luck!*

Answers for the multiple choice questions should be written in this table.

| Question | Answer |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |

[4 pt.] 1. There is 20% chance that an anti-virus program can detect a particular virus. The program is run 10 independent times. Let $X$ denote the number of times that the program detects the virus.

i) Determine the probability mass function of $X$ at 3, i.e. $f_X(3)$.

$X \sim binom(10, 0.2)$

$$f_X(3) = \binom{10}{3}(0.2)^3(0.8)^7 = 0.20133$$

ii) What is the mean number of times that the program detects the viurs?

$$E(X) = np = 10(0.2) = 2$$

iii) Determine the standard deviation of $X$.

$$V(X) = np(1-p) = 2(0.8) = 1.6$$

$$\sigma_X = \sqrt{1.6} = 1.2649$$

iv) Determine the probability that the program detects the virus at most 2 times.

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \binom{10}{0}(0.2)^0(0.8)^{10} + \binom{10}{1}(0.2)^1(0.8)^9 + \binom{10}{2}(0.2)^2(0.8)^9$$

$$\approx 0.68$$

[3 pt.] Morse code uses "dots" and "dashes", which are known to occur in the proportion 3:4, i.e. if $D$ determines the event that "a dot is sent", we have $P(D) = 3/7$. When coded messages are sent, there can be errors in the transmission. Suppose that with probability 1/8, a dot sent is erroneously received as a dash and with probability 1/8 a dash sent is erroneously received as a dot.

i) What is the probability of receiving a dot?

$D$: a dot is sent $RD$: a dot is received

$D^c$: a dash " " $(RD)^c$: a dash is received

$$P(RD) = P(RD|D)P(D) + P(RD|D^c)P(D^c)$$

$$= \frac{7}{8}\frac{3}{7} + \frac{1}{8}\frac{4}{7} = \frac{21}{56} + \frac{4}{56} = \frac{25}{56} \approx 0.45$$

ii) If a dot is received, what is the probability that a dot was sent?

$$P(D|RD) = \frac{P(RD|D)P(D)}{P(RD)} = \frac{\frac{7}{8}\frac{3}{7}}{25/56} = \frac{21}{25} \approx 0.84$$

**Multiple Choice Questions, 2 points each.**

**Submit your answers for the multiple choice questions in the table found on the second page.**

**Question 1.** How many different words (meaningful or meaningless) can be formed using all letters in the word "bookkeeper"?

(A) 1

(B) 10!

(C) $\frac{10!}{2!\,2!\,3!}$

(D) 12!

(E) $\binom{10}{7}$

b : 1
o : 2
k : 2
e : 3
p : 1
r : 1
Total = 10

General permutation

$$\frac{10!}{1!\,2!\,2!\,3!\,1!\,1!}$$

**Question 2.** There are 15 female and 1 male students in a classroom. If a sample of 10 students is chosen randomly, what is the probability that the male student is not in the sample?

(A) $\frac{6}{16}$

(B) $\frac{9}{16}$

(C) $\frac{9}{25}$

(D) $\frac{16}{25}$

(E) $\frac{10}{16}$

$$\frac{\binom{15}{10}}{\binom{16}{10}} = \frac{\frac{15!}{10!\,5!}}{\frac{16!}{10!\,6!}} = \frac{6}{16}$$

**Question 3.** If $A$, $B$ and $C$ are mutually exclusive events with $P(A) = 0.1$, $P(B) = 0.3$ and $P(C) = 0.2$, determine $P(A' \cap B' \cap C')$.

(A) 0.006

(B) 0.504

(C) 0.6

(D) 0.4

(E) None of the preceding.

$$P(A' \cap B' \cap C') = P((A \cup B \cup C)') = 1 - P(A \cup B \cup C)$$
$$= 1 - [P(A) + P(B) + P(C)] = 1 - [0.1 + 0.3 + 0.2] = 0.4$$

**Question 4.** A lot of 20 semiconductor chips contains 3 that are defective. Semiconductors are chosen randomly one by one and without replacement. What is the probability that the first and the second chosen semiconductors are defective?

(A) 0.016

(B) 0.0225

(C) 0.15

(D) 0.001

(E) 0.895

$D_i$: $i$th chip is defective

$$P(D_1 \cap D_2) = P(D_2 | D_1) P(D_1)$$
$$= \frac{2}{19} \frac{3}{20} \approx 0.016$$

**Question 5.** Probability mass function of $X$ is given as

$$f_X(x) = \frac{3x}{18},$$

where $x = 1, 2, 3$. Determine $E(X)$.

(A) 1.78

(B) 2

(C) 3

(D) 3/18

(E) 2.33

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $f(x)$ | $3/18$ | $6/18$ | $9/18$ |

$$E(X) = (1)\frac{3}{18} + (2)\frac{6}{18} + (3)\frac{9}{18} \approx 2.33$$

**Question 6.** Suppose that $E(X) = 5, V(X) = 5, E(Y) = 4, V(Y) = 1$. If $X$ and $Y$ are independent, evaluate standard deviation of $X + 2Y$, i.e. $\sigma_{X+2Y}$.

(A) 6

(B) 7

(C) 8

(D) 3

(E) 4

Because $X$ and $Y$ are independent

$$V(X+2Y) = V(X) + 4V(Y)$$

$$= 5 + 4(1) = 9$$

$$\sigma_{X+2Y} = \sqrt{9} = 3$$

**Question 7.** In a clinical study, volunteers are tested for a gene that has been found to increase the risk for a disease. The probability that a person carries the gene is 0.3. If the gene is not detected in the first 100 volunteers, determine the probability that the 104th volunteer is the first volunteer that carries the gene.

(A) 10.29%

(B) 1.89%

(C) 0.81%

(D) 12.34%

(E) 15.45%

$X$: number of tests until first detection

$X$ is geometric with parameter 0.3

$$P(X=104 \mid X>100) = P(X=4)$$

$$= (0.7)^3 (0.3) = 0.1029$$

**Question 8.** Flaws occur in the interior of plastic used for snowmobiles according to a Poisson distribution with a mean of 0.02 flaw per panel. Determine the probability of at least one flaw in a panel.

(A) 12%

(B) 2%

(C) 44.89%

(D) 98.02%

(E) 1.98%

$X$: number of flaws in a panel

$X \sim \text{Poisson}(0.02)$

$$P(X \geq 1) = 1 - P(X<1) = 1 - P(X=0)$$

$$= 1 - e^{-0.02} \frac{0.02^0}{0!} \approx 0.0198$$

**Question 9.** The number of failures of an instrument is a Poisson random variable with a mean of 0.5 failures per hour. If 10 independent instruments are tested for a period of one hour, what is the probability that exactly two instruments fail during the testing period?

(A) 0.13

(B) 0.16

(C) 0.15

(D) 0.18

(E) 0.17

X: # of failures per instrument

$X \sim$ Poisson$(0.5)$

P(failure for one instrument) =

$$P(X \geq 1) = 1 - P(X=0) = 1 - e^{-0.5} \approx 0.3935$$

Y: # of instruments among 10 that fail

$Y \sim$ binom$(10, 0.3935)$

$$P(Y=2) = \binom{10}{2}(0.3935)^2(1-0.3935)^8 = 0.12757$$

$$\approx 0.13$$