CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number		Section(s)
Mathematics	208/2		All except EC
Examination	**Date**	**Time**	**Pages**
Final	December 2011	3 Hours	3
Instructors			**Course Examiner**
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FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\,\frac{(1+i)^n - 1}{i}, \quad PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- Answer all questions.
- **Only approved calculators are allowed.**

MARKS

[10] **1.** Given the quadratic function $f(x) = 1.2 + 0.96x - 0.12x^2$

(A) Find x and y intercepts algebraically.

(B) Find the vertex form of f.

(C) Find the vertex and the maximum or minimum.

(D) Find the range of f.

[10] **2.** Solve for x in the following equations:

(A) $\left(\frac{3}{4}\right)^x = \frac{16}{9}$

(B) $(0.5)^{-3x^2+15x-72} = (0.5)^{-x^2+35x-22}$

(C) $\log_3\left(\frac{x}{5}\right) + \log_3 7 + 2\log_3\sqrt{5} = 3\log_3\sqrt[3]{175} + 5\log_3 1$

(D) $\log_a x + \log_a(x+1) = \log_a 6$

(E) $\log_2(\sqrt{2x^2}) - 1 = \frac{3}{2}$

[10] **3.** For $f(x) = -12x + 16$ and $g(x) = 3(0.8)^x$ find the following:

(A) $\sum_{k=0}^{49} f(k) = f(0) + f(1) + f(2) + \cdots + f(49)$.

(B) $\sum_{h=0}^{24} g(h) = g(0) + g(1) + g(2) + \cdots + g(24)$.

[10] **4.** Kelly sells some land in Quebec. She will be paid a lump sum of \$60,000 in 7 years. Until then, the buyer pays 8% simple interest quarterly.

(A) Find the amount of each quarterly interest payment.

(B) The buyer sets up a sinking fund so that enough money will be present to pay off the \$60,000. The buyer wants to make semiannual payments into the sinking fund; the account pays 6% compounded semiannually. Find the amount of each payment into the fund.

(C) What is the amount in the sinking fund after the first two deposits.

[10] **5.** A person purchased a house 10 years ago for \$160,000. The house was financed by paying 20% down and signing a 30−year mortgage at 7.75% on the unpaid balance with payments made monthly.

(A) What is the unpaid balance after 120th payment?

(B) After the 120th payment, the owner wishes to refinance the house due to a need for additional cash. If the loan company agrees to a new 30−year mortgage of 80% of the new appraised value of the house, which is \$225,000, how much cash will the owner receive after repaying the balance of the original mortgage?

[10] **6.** Solve by using Gauss-Jordan Elimination:

$$2x_1 + 6x_2 + 15x_3 = -12$$

$$4x_1 + 7x_2 + 13x_3 = -10$$

$$3x_1 + 6x_2 + 12x_3 = -9$$

No other method of solving these systems of equations will be accepted!

[10] **7.** An economy is based on three sectors, agriculture, energy, and manufacturing. Production of a dollar's worth of agriculture requires an input of \$0.20 from the agriculture sector and \$0.40 from the energy sector. Production of a dollar's worth of energy requires an input of \$0.20 from the energy sector and \$0.40 from the manufacturing sector. Production of a dollar's worth of manufacturing requires an input of \$0.10 from the agriculture sector, \$0.10 from the energy sector, and \$0.30 from the manufacturing sector.

(A) Write the technological matrix M for this economy.

(B) If a final demand of \$20 billion for agriculture, \$10 billion for energy, and \$30 billion for manufacturing is to be met, then set up the equation to be satisfied by the inputs from the respective sectors.

(C) Solve the respective inputs satisfying these demands.

[10] **8.** Extremize $P(x, y) = 30x + 10y$ subject to

$$2x + 2y \geq 4,\ 6x + 4y \leq 36,\ 2x + y \leq 10,\ x \geq 0,\ y \geq 0.$$

[10] **9.** A small town has two radio stations: an AM station and an FM station. A survey of 100 town residents produced the following results: In the last 30 days, 65 people have listened to the AM station, 45 have listened to the FM station, and 30 have listened to both stations. During this 30–day period,

(A) How many people in the survey have listened to the AM station but not to the FM station?

(B) How many people in the survey have listened to the FM station but not to the AM station?

(C) How many people in the survey have not listened to either station?

[10] **10.** A study on body types gave the following results: 45% were short, 25% were short and overweight, and 24% were tall and not overweight. Find the probabilities that a person is the following:

(A) Overweight

(B) Short, but not overweight

(C) Tall and overweight

MATH 209 DEC 2011 Final Exam

1 (A) $f(x) = 0 \Rightarrow 0.12x^2 - 0.96x - 1.2 = 0$

$x^2 - 8x - 10 = 0 \Rightarrow x = \frac{8 \pm \sqrt{104}}{2}$

$x = 4 \pm \sqrt{26}$: x intercept

y intercept: $y = -1.2$

(B) $y = -0.12(x^2 - 8x + 16) + 1.2 + 1.92$

$= -0.12(x-4)^2 + 3.12$

(C) Vertex $(4, 3.12)$, $f(4) = 3.12$ is max. at $x = 4$.

(D) Range: $[3.12, \infty)$

2 (A) $\left(\frac{3}{4}\right)^x = \left(\frac{4}{3}\right)^2$ or $\left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^{-2} \Rightarrow x = -2$

ANS: $x = -2$

(B) $-3x^2 + 15x - 72 = -x^2 + 35x - 22$

$2x^2 + 20x + 50 = 0$

$x^2 + 10x + 25 = 0$

$(x+5)^2 = 0 \Rightarrow x = -5, -5$

(C) $\log_3\left[\frac{x}{5} \cdot 7 \cdot 5\right] = \log_3 175 + 0$

$7x = 175 \Rightarrow x = 25$

(D) $\log_a x(x+1) = \log_a 6$

$x(x+1) = 6 \Rightarrow x^2 + x - 6 = 0$

$(x+3)(x-2) = 0 \Rightarrow x = 2, x = -3$

$x = -3$ is not acceptable or possible.

So answer is: $x = 2$

(E) $\frac{1}{2}\log_2(2x^2) = \frac{5}{2} \Rightarrow \log_2(2x^2) = 5$

$2x^2 = 2^5 \Rightarrow 2x^2 = 32 \Rightarrow x = \sqrt{16} = 4$

3 (A) $\sum_{K=0}^{49} f(K) = 16 + 4 - 8 - \cdots - 572$

$= \frac{50}{2}[16 - 572] = -13900$

(B) $\sum_{K=0}^{24} 3(0.8)^K = 3 + 3(0.8) + 3(0.8)^2 + \cdots + 3(0.8)^{24}$

$= \frac{3[1 - (0.8)^{25}]}{1 - 0.8} = 14.94333$

4 (A) Quarterly simple interest = 2%

Total quarterly interest = $60,000 \times .02$

(B) Interest per period = \$1200/quarter

$i = \frac{.06}{2} = 0.03$. Future Value = 60,000

$FV = \frac{PMT[(1+i)^n - 1]}{i} \Rightarrow PMT = \frac{60000 \times .03}{(1.03)^{14} - 1}$

PMT = \$3511.58 semiannual PMT

(C) $FV = R + 12(1+i)$

$= 3511.58[2.03] = \$7128.51$

5 Loan: $160,000 - 32000 = \$128,000$

Periodic interest $i = \frac{0.0775}{12} = 0.00645833$

$PV = PMT\frac{[1 - (1+i)^{-n}]}{i} \rightarrow n = 30 \times 12 = 360$

$PMT = \frac{128000 \times 0.006458}{1 - (1.0064583)^{-360}} = \$917/\text{month}$

(A) Unpaid balance after 120 payments

$= \frac{R[1 - (1+i)^{-60-120}}{i} = \frac{917[1 - (1.006458)^{-240}]}{0.00645833}$

$= \$111,700.06$

(B) New loan amount $= 225,000 \times 0.8$

$= \$180,000$

Cash received $= 180,000 - 111,700.06$

$= \$68,299.94$

6 Augmented matrix:

$$\begin{bmatrix} 2 & 6 & 15 & -12 \\ 4 & 7 & 13 & -10 \\ -3 & 6 & 12 & -9 \end{bmatrix} \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \begin{bmatrix} 2 & 6 & 15 & -12 \\ 0 & -5 & -17 & 14 \\ 1 & 0 & -3 & 3 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3}$$

$$\begin{bmatrix} 1 & 0 & -3 & 3 \\ 0 & -5 & -17 & 14 \\ 2 & 6 & 15 & -12 \end{bmatrix} \xrightarrow{R_3 - 2R_1} \begin{bmatrix} 1 & 0 & -3 & 3 \\ 0 & -5 & -17 & 14 \\ 0 & 6 & 21 & -18 \end{bmatrix} \xrightarrow{R_3 + R_2}$$

$$\begin{bmatrix} 1 & 0 & -3 & 3 \\ 0 & -5 & -17 & 14 \\ 0 & 1 & 4 & -4 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 0 & -3 & 3 \\ 0 & 1 & 4 & -4 \\ 0 & -5 & -17 & 14 \end{bmatrix} \xrightarrow{R_3 + 5R_2}$$

$$\begin{bmatrix} 1 & 0 & -3 & 3 \\ 0 & 1 & 4 & -4 \\ 0 & 0 & 3 & -6 \end{bmatrix} \xrightarrow{\frac{1}{3}R_3} \begin{bmatrix} 1 & 0 & -3 & 3 \\ 0 & 1 & 4 & -4 \\ 0 & 0 & 1 & -2 \end{bmatrix} \xrightarrow[R_2 - 4R_3]{R_1 + 3R_3}$$

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{bmatrix} \Rightarrow \begin{cases} x_1 = -3 \\ x_2 = 4 \\ x_3 = -2 \end{cases}$$ Ans

7 (A) Technology Matrix

$$M = \begin{bmatrix} 0.2 & 0 & .1 \\ 0.4 & 0.2 & .1 \\ 0 & .4 & .3 \end{bmatrix}$$

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Let x : Total Agri. Produ required
y : Total Energy " "
z : Total Manufacturing required

$x = 0.2x + 0.1z + 20$
$y = 0.4x + 0.2y + 0.1z + 10$
$z = 0.4y + 0.3z + 30$

OR

$0.8x - 0.1z = 20$
$-0.4x + 0.8y - 0.1z = 10$
$-0.4y + 0.7z = 30$

} Solve This system

Start with Augmented matrix

$$\begin{bmatrix} 8 & 0 & -1 & 200 \\ -4 & 8 & -1 & 100 \\ 0 & -4 & 7 & 300 \end{bmatrix} \xrightarrow{R_1+2R_2}$$

$$\begin{bmatrix} 0 & 16 & -3 & 400 \\ -4 & 8 & -1 & 100 \\ 0 & -4 & 7 & 300 \end{bmatrix} \xrightarrow[R_2+2R_3]{R_1+4R_2} \begin{bmatrix} 0 & 0 & 25 & 1600 \\ -4 & 0 & 13 & 700 \\ 0 & -4 & 7 & 300 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{25}R_1} \begin{bmatrix} 0 & 0 & 1 & 64 \\ -4 & 0 & 13 & 700 \\ 0 & -4 & 7 & 300 \end{bmatrix} \xrightarrow[R_3-7R_1]{R_2+13R_1}$$

$$\begin{bmatrix} 0 & 0 & 1 & 64 \\ -4 & 0 & 0 & -132 \\ 0 & -4 & 0 & -148 \end{bmatrix} \xrightarrow[-\frac{1}{4}R_3]{-\frac{1}{4}R_2} \begin{bmatrix} 0 & 0 & 1 & 64 \\ 1 & 0 & 0 & 33 \\ 0 & 1 & 0 & 37 \end{bmatrix}$$

$\therefore$ $x = 33$ Billion $
$y = 37$ " "
$z = 64$ " "

} Ans

Note: See page 241 Example 1 — Textbook →

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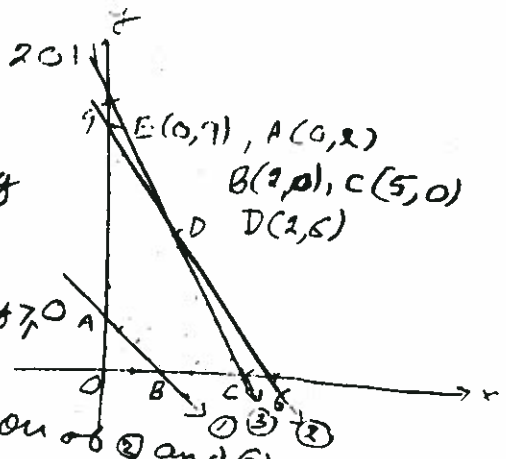
$P(x,y) = 30x + 10y$
$2x + 2y \geq 4$ ①
$6x + 4y \leq 36$ ②
$2x + y \leq 10$, $x, y \geq 0$



D: is intersection of ② and ③

$3x + 2y = 18$
$2x + y = 10$ $\Rightarrow$ $3x + 2y = 18$
$-4x - 2y = -20$
$-x = -2 \Rightarrow x = 2$
$y = 6$

Now evaluate $P(x,y) = 30x + 10y$

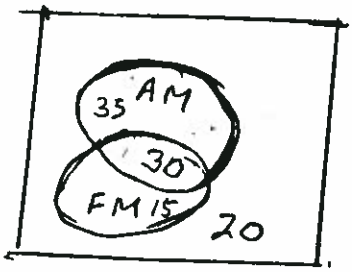
AE : A(0,2) : $P(0,2) = 20$
B(2,0) $P(2,0) = 60$
C(5,0) $P(5,0) = 150$
D(2,6) $P(2,6) = 60 + 60 = 120$
E(0,9) $P(0,9) = 90$

Max $P = 150$ at $x = 5$, $y = 0$
Min $P = 20$ at $x = 0$, $y = 2$

⑨ Total = 100

Ⓐ Ans: 35
Ⓑ Ans: 15
Ⓒ Ans: 20

} See



10 See the (completed) table below.

	SHORT	TALL	Total
OVER-weight	25	(31)	(56)
NOT OVER-weight	(20)	24	(44)
Total	45	(55)	100

Ⓐ $\frac{56}{100} = 0.56$

Ⓑ $\frac{20}{100} = 0.2$

Ⓒ $\frac{31}{100} = 0.31$

Given

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$$\left.\begin{aligned} 0.7x - 0.3y - 0.10z &= 25 \\ -0.1x + 0.9y - 0.10z &= 15 \\ -0.2x - 0.20y + 0.9z &= 20 \end{aligned}\right\} \times 10 \Rightarrow \begin{aligned} 7x - 3y - z &= 250 \\ -x + 9y - z &= 150 \\ -2x - 2y + 9z &= 200 \end{aligned}$$

Solve the Augmented matrix of the system

$$\left[\begin{array}{ccc|c} 7 & -3 & -1 & 250 \\ -1 & 9 & -1 & 150 \\ -2 & -2 & 9 & 200 \end{array}\right] \xrightarrow[R_2 \leftrightarrow R_1]{-1\,R_2} \left[\begin{array}{ccc|c} 1 & -9 & 1 & -150 \\ 7 & -3 & -1 & 250 \\ -2 & -2 & 9 & 200 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -9 & 1 & -150 \\ -2 & -2 & 9 & 200 \\ 7 & -3 & -1 & 250 \end{array}\right]$$

$$\xrightarrow[R_3 - 7R_1]{R_2 + 2R_1} \left[\begin{array}{ccc|c} 1 & -9 & 1 & -150 \\ 0 & -20 & 11 & -100 \\ 0 & 60 & -8 & 1300 \end{array}\right] \xrightarrow[3R_2 + R_3]{\frac{R_2}{-20}} \left[\begin{array}{ccc|c} 1 & -9 & 1 & -150 \\ 0 & 1 & \frac{-11}{20} & 5 \\ 0 & 0 & 25 & 1000 \end{array}\right] \xrightarrow{R_1 + 9R_2} \left[\begin{array}{ccc|c} 1 & 0 & \frac{-79}{20} & -105 \\ 0 & 1 & \frac{-11}{20} & 5 \\ 0 & 0 & 25 & 1000 \end{array}\right]$$

$$\xrightarrow{\frac{R_3}{25}} \left[\begin{array}{ccc|c} 1 & 0 & \frac{-79}{20} & -105 \\ 0 & 1 & \frac{-11}{20} & 5 \\ 0 & 0 & 1 & 40 \end{array}\right] \xrightarrow[R_2 + \frac{11}{20}R_3]{R_1 + \frac{79}{20}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 53 \\ 0 & 1 & 0 & 27 \\ 0 & 0 & 1 & 40 \end{array}\right]$$

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