CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number		Section(s)
Mathematics	208/2		All except EC
Examination	**Date**	**Time**	**Pages**
Final	December 2010	3 Hours	3

Instructors — **Course Examiner**

A. Kokotov, B. Rhodes, E. Duma, E. Smith, J. McSweeney, J. Ruddy, R. Perez-Buendia, T. Koulis, U. Tiwari — D. Sen

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\,\frac{(1+i)^n - 1}{i}, \quad PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- ▷ Answer all questions.
- ▷ **Only approved calculators are allowed.**

MARKS

[10] **1.** Given the quadratic function $f(x) = -0.15x^2 - 0.90x + 3.3$ $a = -0.15,\ b = -0.90,\ c = 3.3$

(A) Find x and y intercepts algebraically. $x = -8.568, 2.568,\ y = 3.3$

(B) Find the vertex form of f. $\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right) = (-3, 4.65)$ $-0.15(x+3)^2 + 4.65$

(C) Find the vertex and the maximum or minimum. $(-3, 4.65)$ max $= 4.65$

(D) Find the range of f. $(-\infty, 4.65]$

[10] **2.** Solve for x in the following equations:

(A) $4^{x^2-4x} = \dfrac{1}{256}$

(B) $(100)^{2x} = (10)^{x^2-12}$ $\quad 4x = x^2 - 12 \Rightarrow x^2 - 4x - 12 \Rightarrow (x-6)$

$x = -2, 6$

(C) $3\log_2(x-1) + \log_2 4 = 5$ $\quad \Rightarrow x = 3$

(D) $\log_a x + \log_a(x-4) = \log_a(x+6)$ $\quad x(x-4) = x+6 \Rightarrow (x-6)(x+1) = 0$, $x = -1$ No good

$x = 6$

(E) $\log_3(4x-7) = 2$

$4x - 7 = 3^2$

$4x = 16$

$x = 4$

[10] **3.** For $f(x) = 24 - 6x$ and $g(x) = 5^{x-5}$ find the following:

(A) $\sum_{k=0}^{49} f(k) = f(0) + f(1) + f(2) + \cdots + f(49)$.

$= 24 + 18 + 12 \quad -270 \quad S_{50} = \frac{n}{2}[a_0 + a_{49}]$

$= -6150$

(B) $\sum_{h=0}^{29} g(h) = g(0) + g(1) + g(2) + \cdots + g(29)$.

$= 5^{-5} + 5^{-4} \quad + 5^{24} = \frac{5^{-5}[5^{30}-1]}{5-1} =$

$a_1 = 5^{-5}, \; r = 5$

[10] **4.** A man deposits \$2,000 in an IRA on his 21st birthday and on each subsequent birthday up to, and including, his 29th. The account earns 8% compounded annually. Future value of deposit on 29th b day $= \frac{2000[(1.08)^9-1]}{0.08} = \$24,975.12$

(A) If he leaves the money in the account without making any more deposits, how much will he have on his 65th birthday, assuming the account continues to earn the same rate of interest? $A = P(1+i)^n = (24975.12)(1.08)^{36} = \$398807$

(B) How much would be in the account on his 65th birthday if he had started the deposits on his 30th birthday and continued making deposits on each birthday untill (and including) his 65th birthday?

$FV = \frac{2000[(1.08)^{36}-1]}{0.08} = \$374,264.30$

[10] **5.** On December 31, 1990, a house was purchased with the buyer taking out a 30 year, \$112,475 mortgage at 9% interest, compounded monthly. The mortgage payments are made at the end of each month. $i = \frac{0.09}{12} = 0.0075, \; n = 360$

Ans PMT = \$905 per month ⇐ (A) Calculate the amount of the monthly payment. $PMT = \frac{(112475)(0.0075)(1.0075)^{360}}{(1.0075)^{360}-1}$

(B) Calculate the unpaid balance of the loan on December 31, 2016. $PV = \frac{905[1 - 1.0075^{-?}]}{0.0075}$

ANS \$71442.23

(C) How much of the principal will be paid off during the year 2016?

unpaid balance on Dec 31, 2015 − unpaid balance on Dec 31, 2016 = \$75663.? − \$71442.23 = \$4221.58

(D) How much interest will be paid during the year 2016?

Ans \$905 × 12 − \$4221.58 = \$6,638.42

[10] **6.** A company that rents small moving trucks wants to purchase 25 trucks with a combined capacity of 28,000 cubic feet. Three different types of trucks are available: a 10-foot truck with a capacity of 350 cubic feet, a 14-foot truck with a capacity of 700 cubic feet, and a 24-foot truck with a capacity of 1,400 cubic feet.

(A) Write the linear system of equations in terms of x, y and z; x, y and z being the number of 10-foot trucks, 14-foot trucks, and 24-foot trucks respectively.

$\begin{cases} x + 2y + 4z = 80 \\ x + y + z = 25 \end{cases}$ general soln $x = -30 + 2t \geq 0$, $y = 55 - 3t \geq 0$, $z = t \geq 0$ $\quad t = 15, 16, 17, 18$

Ans (B) Solve this system of equations.

① $x = 0, y = 10, z = 15$ ② $x = 2, y = 7, z = 16$ ③ $x = 4, y = 4, z = 17$ ④ $x = 6, y = 1, z = 18$

(C) The rental company charges \$19.95 per day for a 10-foot truck, \$29.95 per day for a 14-foot truck, and \$39.95 per day for a 24-foot truck. Which of the solutions would produce the largest daily income from the truck rentals?

Ans ① $19.95 \times 0 + 10 \times 29.95 + 15 \times 39.95 = \$898.75$

② $19.95 \times 2 + 7 \times 29.95 + 16 \times 39.95 = \$888.75$ max revenue = \$898.75

③ $19.95 \times 4 + 4 \times 29.95 + 17 \times 39.95 = \$878.75$ when $x = 0, y = 10, z = 15$

④ $19.95 \times 6 + 1 \times 29.95 + 18 \times 39.95 = \$868.75$

[10] **7.** An economy is based on three sectors, agriculture, manufacturing, and energy. Production of a dollar's worth of agriculture requires an input of \$0.20 from agriculture, \$0.20 from manufacturing, and \$0.20 from energy. Production of a dollar's worth of manufacturing requires an input of \$0.40 from agriculture, \$0.10 from manufacturing, and \$0.10 from energy. Production of a dollar's worth of energy requires an input of \$0.30 from agriculture, \$0.10 from manufacturing, and \$0.10 from energy.

$$M = \begin{array}{c|ccc} & A & M & E \\ \hline A & 0.20 & 0.40 & 0.30 \\ M & 0.20 & 0.10 & 0.10 \\ E & 0.20 & 0.10 & 0.10 \end{array}$$

(A) Write the technological matrix M for this economy.

$I - M, X = D$

$$\begin{bmatrix} 0.80 & -0.40 & -0.30 \\ -0.20 & 0.90 & -0.10 \\ -0.20 & -0.10 & 0.90 \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ 20 \end{bmatrix}$$

(B) If a final demand of \$10 billion for agriculture, \$15 billion for manufacturing, and \$20 billion for energy is to be met, then set up the equation to be satisfied by the inputs from the respective sectors.

(C) Solve the respective inputs satisfying these demands.

Augmented Matrix

$$\begin{bmatrix} 8 & -4 & -3 & 100 \\ -2 & 9 & -1 & 150 \\ -2 & -1 & 9 & 200 \end{bmatrix} \xrightarrow[\text{soln.}]{\text{use EROS To find}} \begin{bmatrix} 1 & 0 & 0 & 40.1 \\ 0 & 1 & 0 & 29.4 \\ 0 & 0 & 1 & 34.4 \end{bmatrix}$$

$x = 40.1$ billion

$y = 29.4$ "

$z = 34.4$ "

[10] **8.** Extremize $P(x, y) = 40x + 20y$ subject to

$$6x + 9y \geq 90,\ 2x + y \leq 26,\ -2x + 5y \leq 34,\ x \geq 0,\ y \geq 0.$$

A(3, 8)

B(12, 2), C(8, 10), max on line BC = 520 max.

Min at A(3, 8) = 280 min

[10] **9.** A software development department consists of 6 women and 4 men.

(A) How many ways can the department select a chief programmer, a backup programmer, and a programming librarian?

(B) How many of the selections in part (A) consist entirely of women?

(C) How many ways can the department select a team of 3 programmers to work on a particular project?

[10] **10.** Six popular brands of cola are to be used in a blind taste study for consumer recognition.

(A) If 3 distinct brands are chosen at random from the 6 and if a consumer is not allowed to repeat any answers, what is the probability that all 3 brands could be identified by just guessing?

(B) If repeats are allowed in the 3 brands chosen at random from the 6 and if a consumer is allowed to repeat answers, what is the probability that all 3 brands are identified correctly by just guessing?

$-2x + 5y = 34$

$2x + y = 26$

$6y = 60$

$y = 10$

[redacted]

March 6th, 2012

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	Ag	Ma	En	D
Ag	0.20	0.40	0.30	$10 B
Ma	0.20	0.10	0.10	$15 B
En	0.20	0.10	0.10	$20 B

<u>Ans</u>

$x = \$40.1$ Billions

$y = \$29.4$ "

$z = \$34.4$ "

$$\begin{cases} x = 0.20x + 0.40y + 0.30z + 10 \\ y = 0.20x + 0.10y + 0.10z + 15 \\ z = 0.20x + 0.10y + 0.10z + 20 \end{cases}$$

<u>Ans</u> $x = 40.1$ Billion

$y = 29.4$ Bill

$z = 34.4$ Bill

$$\begin{cases} 0.8x - 0.4y - 0.3z = 10 \quad (10) \\ -0.2x + 0.9y - 0.1z = 15 \quad (10) \\ -0.2x - 0.1y + 0.9z = 20 \quad (10) \end{cases}$$

$$\left[\begin{array}{ccc|c} 8 & -4 & -3 & 100 \\ -2 & 9 & -1 & 150 \\ -2 & -1 & 9 & 200 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} -2 & 9 & -1 & 150 \\ 8 & -4 & -3 & 100 \\ -2 & -1 & 9 & 200 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 + 4R_1} \left[\begin{array}{ccc|c} -2 & 9 & -1 & 150 \\ 0 & 32 & -7 & 700 \\ 0 & -10 & 10 & 50 \end{array}\right]$$

$$\downarrow -\tfrac{1}{10}R_3$$

$$\left[\begin{array}{ccc|c} -2 & 0 & 8 & 195 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 25 & 860 \end{array}\right] \xleftarrow[R_3 - 32R_2]{R_1 - 9R_2} \left[\begin{array}{ccc|c} -2 & 9 & -1 & 150 \\ 0 & 1 & -1 & -5 \\ 0 & 32 & -7 & 700 \end{array}\right] \xleftarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} -2 & 9 & -1 & 150 \\ 0 & 32 & -7 & 700 \\ 0 & 1 & -1 & -5 \end{array}\right]$$

$$\downarrow \tfrac{1}{25}R_3$$

$$\left[\begin{array}{ccc|c} -2 & 0 & 8 & 195 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 1 & 172/5 \end{array}\right] \xrightarrow[R_2 + R_3]{R_1 - 8R_3} \left[\begin{array}{ccc|c} -2 & 0 & 0 & -401/5 \\ 0 & 1 & 0 & 147/5 \\ 0 & 0 & 1 & 172/5 \end{array}\right] \xrightarrow{-\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 401/10 \\ 0 & 1 & 0 & 147/5 \\ 0 & 0 & 1 & 172/5 \end{array}\right]$$

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Course	**Number**		**Section(s)**
Mathematics	208/2		All
Examination	**Date**	**Time**	**Pages**
Midterm	October 2012	1 Hour 30 minutes	2
Instructors			**Course Examiner**
B. Rhodes, D. Sen, E. Duma, E. Kritchevski, F. Romanelli, K. Benchekroun, P. Oyono Ngou, U. Tiwari			D. Sen

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\frac{(1+i)^n - 1}{i}, \quad PV = PMT\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

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MARKS

[10] **1.** A charter fishing company buys a new boat for \$224,000 and assumes that it will have a trade in value of \$115,200 after 16 years. $t > 18.24$ yrs

(A) Find a linear model for the depreciated value V of the boat t years after it was purchased. $m = -6800$

(B) What is the depreciated value of the boat after 10 years? $V = \$156,00$

(C) When will the depreciated value fall below \$100,000? $t \geq 18.24$ yr

[10] **2.** Solve for x in the following equations:

(A) $8^{x^2-3} = 2^{x+4}$ $\quad x = 2.255, 1.922$

(B) $\ln 2 + \ln(x+2) - \ln(x-1) = 3\ln 2$ $\quad x = 2$

(C) $e^{4x^2-17x} = e^{-8-5x}$ $\quad x = 2.30, -1.30$

(D) $\log_3(x^2 - x + 6) = 2$

PLEASE TURN OVER

[10] **3.**

(A) If the first and 15th terms of an arithmetic sequence are -5 and 23, respectively, find the 73rd term of the sequence. $a_1 = -5, d = d$

$a_{73} = 139$

(B) If the first and 10th terms of a geometric sequence are 4 and 40, respectively, find the 46th term of the sequence. $a_1 = 4, \; a_{10} = 40$

$a_{46} = 4(10)^5 = 400,000$

[10] **4.** Suppose that after buying a new car you decide to sell your old car to a friend. You accept a 270–day note for \$3,500 at 10% simple interest as payment. (Both principal and interest are paid at the end of 270 days.) Sixty days later you find that you need the money and sell the note to a third party for \$3,550. What annual interest rate will the third party receive for the investment?

$I = 262.50, \quad r = 10.26\%$

[10] **5.** Beginning in January, a person plans to deposit \$100 at the end of each month into an account earning 6% compounded monthly. Each year taxes must be paid on the interest earned during that year. Find the interest earned during each year for the first 3 years. ~~I = \$262~~.50, r ~~= 10.26~~ = ~~10.26~~ %

1^{st} yr = 33.56, Year 2 = \$109.64, Yr 3^{rd} yr = \$190.21

[10] **6.** You are buying a \$10,000 second hand car with a down-payment of \$3,000 and you finance the remaining amount with a 3 year loan at 7.8% compounded weekly.

(A) What are your weekly payments? How much interest in total are you paying? \$50. 3

(B) What is the remaining balance after 2 years? = \$2517.38