

Concordia University
Math 209 Midterm Review Sec 3.1-4.6

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Date: 2014/05/16

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marks

[00] 1. Find the value of the following limit algebraically: $\lim_{x \rightarrow -2} \frac{x^2 - 5x - 14}{x + 2}$

[00] 2. Consider $f(x) = 5x^2 + 1$:

a) Evaluate $f'(x)$ using the definition: $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

b) Using the answer to a), evaluate the value of $f'(2)$

c) Determine the equation of the tangent to $f(x)$ at the point where $x = 2$

[00] 3. Find $f'(x)$ for the following (Do not simplify answers):

a) $f(x) = 4x^3 + 7x - 8 + \sqrt[4]{x}$

b) $f(x) = \frac{4x - 5}{x^2 + 3x}$

[00] 4. Find the derivatives of the following functions (Do not simplify answers):

a) $f(x) = (e^x + x)^3(2 + \ln x)^4$

b) $f(x) = [e^{12x^3 - 9x - 15} + 4 \ln(x^3 + 3x - 1)]^{10}$

[00] 5. Revenue $R(x)$, Cost $C(x)$ are given by $R(x) = 200x - \frac{x^2}{30}$ and $C(x) = 72000 + 60x$

Use differentials to approximate the change in profit as production increases from 1500 to 1510 articles.

[00] 6. Revenue $R(x)$ where x is the number of articles sold is given by $R(x) = \frac{x^3}{3} + 200x$

a) Find the average rate of change in the Revenue as x goes from 20 to 21. Describe in words what this answer represents

b) Find $R'(x)$ at a sales level of 20 articles. Describe in words what this answer represents.

c) Use $R'(20)$ and $R(20)$ to approximate $R(21)$.

d) Find the marginal average Revenue at a sales level of 20 articles. Describe in words what this answer represents.

[00] 7. At what nominal (yearly) rate compounded continuously must money be invested to double in ten years ?

[00] 8. The resale value R (in dollars) of a company car after t years is estimated to be given by $R(t) = 20000(0.86)^t$

a) What is the rate of depreciation after one year ?

b) What is the rate of depreciation after two years ?

c) What is the rate of depreciation after three years ?

..... continued

[00] 9. Consider price p and demand x are related by the equation $x = 300 - (p - 15)^2$:

- a) Find the rate of change of demand compared to price for each price given and then explain in words what each answer means: (i) $p = 1$ (ii) $p = 16$.
- b) Find the rate of change of price compared to demand for each price given and then explain in words what each answer means: (i) $p = 1$ (ii) $p = 16$. (round the answers to the nearest cent)

[00] 10. Consider price p and demand x are related by the equation $2x + xp - p^2 = 104$. If the price is increasing at a rate of 2\$ per month, when the price is at 4\$, find the rate of change of the demand per month at this price.

Answers:

1. -9 , 2. a) $10x$ b) 20 c) $y = 20x - 19$, 3. a) $12x^2 + 7 + \frac{1}{4}x^{-\frac{3}{4}}$. b) $\frac{(x^2+3x)(4)-(4x-5)(2x+3)}{(x^2+3x)^2}$, 4.

a) $f(x) = (e^x + x)^3 4(2 + \ln x)^3 \frac{1}{x} + (2 + \ln x)^4 3(e^x + x)^2 (e^x + 1)$. b)

$10[e^{12x^3-9x-15} + 4 \ln(x^3 + 3x - 1)]^9 [e^{12x^3-9x-15}(36x^2 - 9) + \frac{4(3x^2+3)}{x^3+3x-1}]$, 5. 400\$, 6. a) 620.33\$, the exact

Revenue from the 21st article sold. b) 600\$, the marginal revenue when $x = 20$ or the approximate Revenue from the 21st article sold. c) $R(20) + R'(20) = 7266.66$ \$ d) 13.33\$ the increase in Revenue attributed to each article sold by selling the 21st article., 7. 6.93%, 8.

2594\$/year, 2231\$/year, 1919\$/year., 9 a)(i) Demand increasing 28 articles per 1\$ increase in price (at this point price increase is not lowering the demand). (ii) Demand decreasing 2 articles per 1\$ increase in price (at this point price increase is lowering the demand). 9 b) (i) Price increases 0.04\$ per 1 article increase in demand (at this point demand increase is causing the price to rise) (ii) Price decreases 0.50\$ per 1 article increase in demand (at this point demand increase is not causing the price to rise). 10. $\frac{dx}{dt} = -4$ (decrease of 4 articles per month)