7.5 Use the differential equation approach to find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.5.

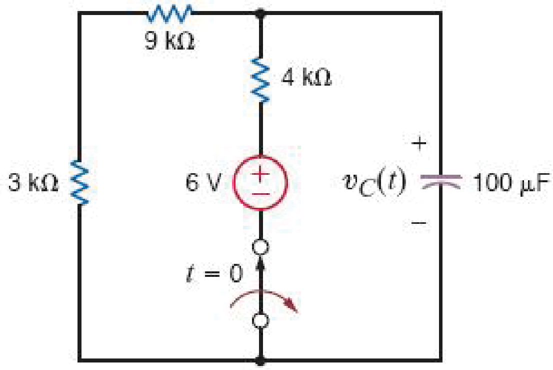
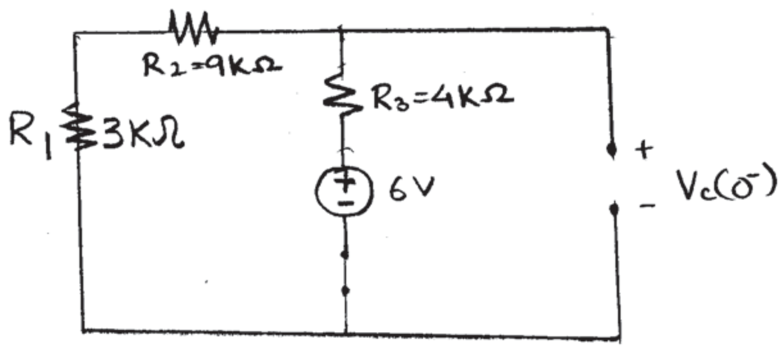


Figure P7.5

SOLUTION:

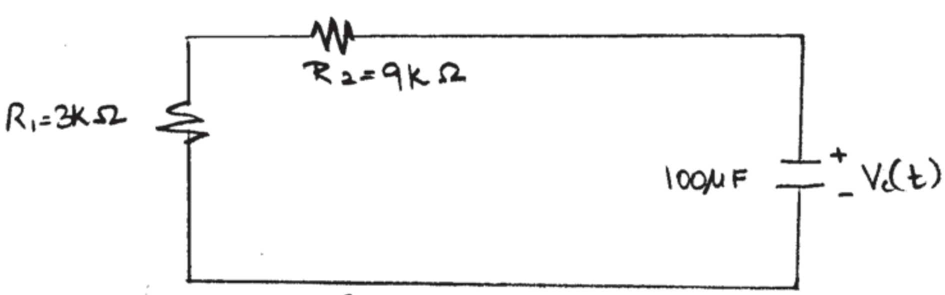
for $t < 0^-$:



$$V_c(0^-) = \left(\frac{R_1 + R_2}{R_1 + R_2 + R_3}\right)(6)$$

$$V_c(0^-) = 4.5\,V$$

for $t > 0$:



$$C\frac{dV_c(t)}{dt} + \frac{V_c(t)}{R_1 + R_2} = 0$$

$$\frac{dV_c(t)}{dt} + \frac{V_c(t)}{(R_1 + R_2)C} = 0$$

$$r + \frac{1}{(R_1 + R_2)C} = 0$$

$$r = \frac{-1}{(R_1 + R_2)C}$$

$$V_c(t) = Ae^{\gamma t}$$

$$V_c(t) = Ae^{-\frac{t}{(R_1 + R_2)C}}$$

$$V_c(0^-) = 4.5V = V_c(0^+)$$

$$A = 4.5$$

$$V_c(t) = 4.5e^{-\frac{t}{1.2}} \text{ V}, \quad t > 0$$

7.7 Use the differential equation approach to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.7 and plot the response including the time interval just prior to opening the switch.

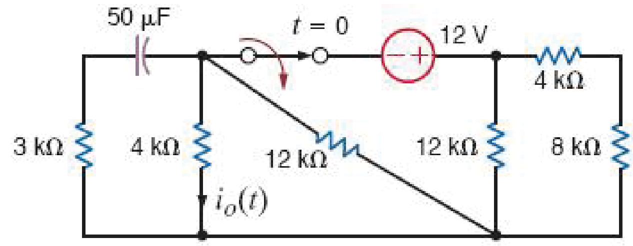
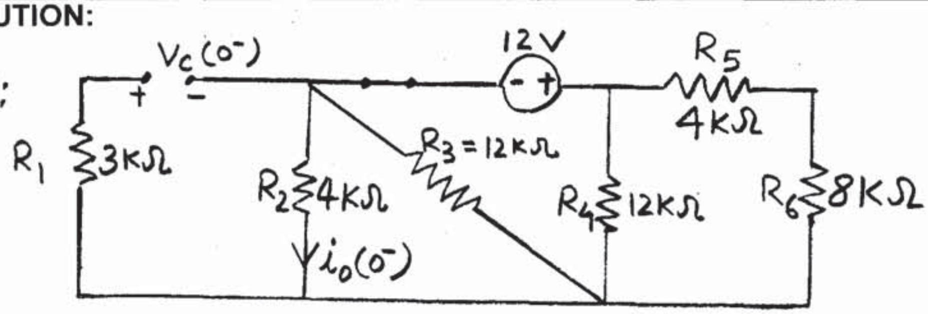


Figure P7.7

SOLUTION:

for $t=0^-$:



$$R_x = R_4 \parallel (R_5 + R_6) = 6\,K\Omega$$

$$R_y = R_2 \parallel R_3 = 3\,K\Omega$$

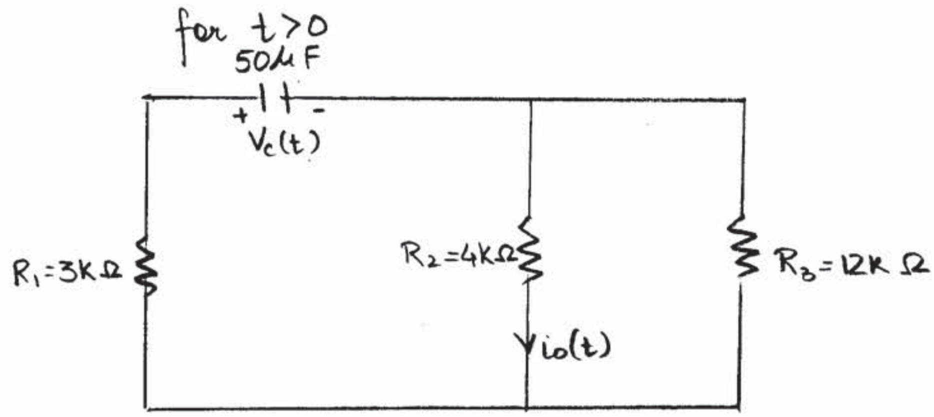
$$V_c(0^-) = \left(\frac{R_y}{R_x + R_y}\right)(12)$$

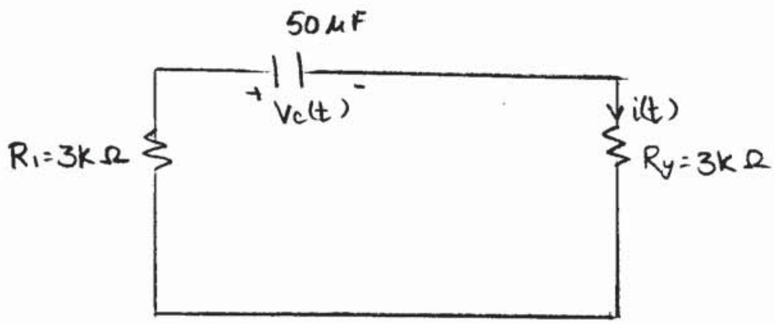
$$V_c(0^-) = 4V$$

$$i_o(0^-) = -\frac{V_c(0^-)}{R_2}$$

$$= -\frac{4}{4K}$$

$$i_o(0^-) = -1\,mA$$





KVL:

$$V_c(t) + R_y i(t) + R_1 i(t) = 0$$

$$i(t) = C\frac{dV_c(t)}{dt}$$

$$R_y C\frac{dV_c(t)}{dt} + R_1 C\frac{dV_c(t)}{dt} + V_c(t) = 0$$

$$\frac{dV_c(t)}{dt} + \frac{V_c(t)}{(R_1+R_y)C} = 0$$

$$\gamma + \frac{1}{(R_1+R_y)C} = 0$$

$$\gamma = -\frac{1}{(R_1+R_y)C}$$

$$V_c(t) = Ae^{\gamma t}$$

$$V_c(t) = A e^{\frac{-t}{(R_1+R_y)C}}$$

$$V_c(t) = A e^{\frac{-t}{0.3}}$$

$$V_c(0^-) = 4V = V_c(0^+)$$

$$A = 4$$

$$V_c(t) = 4e^{\frac{-t}{0.3}} \; V, \; t>0$$

KVL:

$$R_2 i_o(t) + R_1 C \frac{dV_c(t)}{dt} + V_c(t) = 0$$

$$i_o(t) = \frac{-V_c(t) - R_1 C \frac{dV_c(t)}{dt}}{R_2}$$

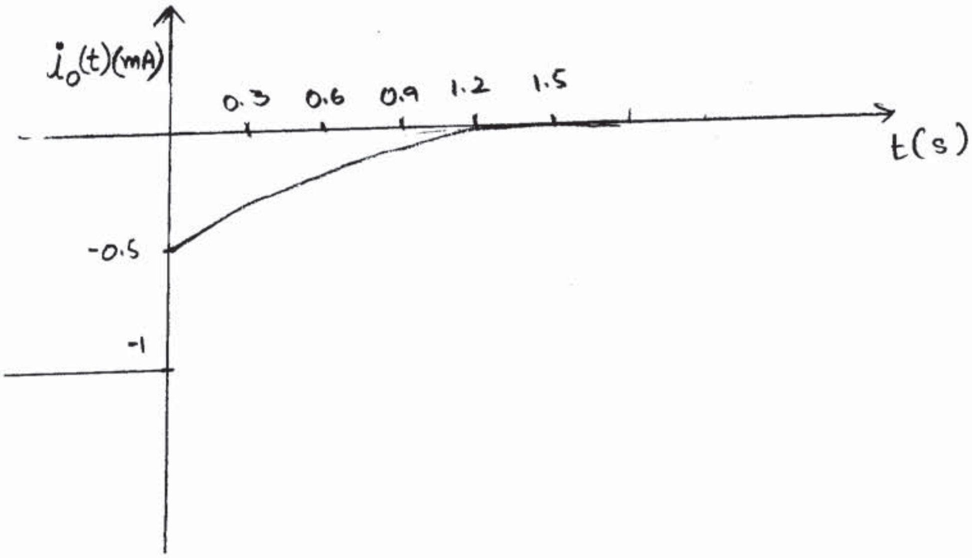
$$\frac{dV_c(t)}{dt} = -13.333 e^{-t/0.3}$$

$$i_o(t) = \frac{-4e^{-\frac{t}{0.3}} - (3\times10^3)(50\times10^{-6})(-13.333 e^{-\frac{t}{0.3}})}{4\times10^3}$$

$$i_o(t) = \frac{-2e^{\frac{-t}{0.3}}}{4\times10^3}$$

$$i_o(t) = -0.5 e^{-\frac{t}{0.3}} \; mA, \; t>0$$

$i_o(0^-) = -1mA$



7.10 In the network in Fig. P7.10, find $i_o(t)$ for $t > 0$ using the differential equation approach.

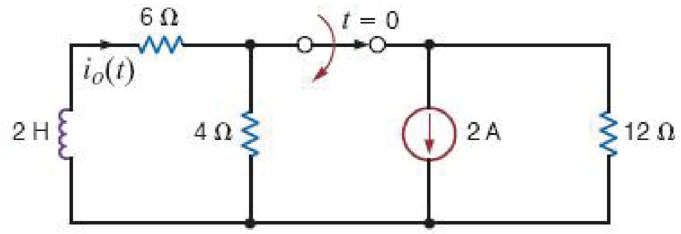
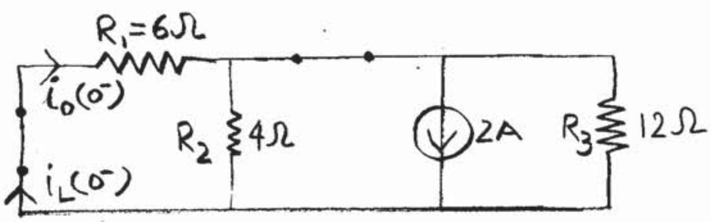


Figure P7.10

SOLUTION: for $t = 0^-$:



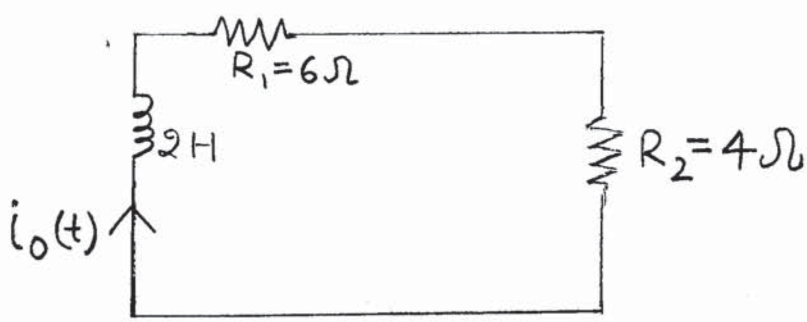
$$R' = R_2 \parallel R_3 = 3\Omega$$

$$i_L(0^-) = \left(\frac{R'}{R_1 + R'}\right)(2)$$

$$i_L(0^-) = \frac{2}{3}A$$

$$i_o(0^+) = i_L(0^+) = \frac{2}{3}A$$

for $t > 0$



$$i_L(t) = i_o(t)$$

KVL: $$L\frac{di_o(t)}{dt} + R_1 i_o(t) + R_2 i_o(t) = 0$$

$$\frac{di_o(t)}{dt} + \frac{R_1+R_2}{L} i_o(t) = 0$$

$$r + \frac{R_1+R_2}{L} = 0$$

$$r = \frac{-(R_1+R_2)}{L}$$

$$i_o(t) = Ae^{rt}$$

$$i_o(t) = A\,e^{-\left(\frac{R_1+R_2}{L}\right)t}$$

$$i_o(t) = Ae^{-5t}$$

$$i_o(0^-) = \frac{2}{3}A = i_o(0^+)$$

$$A = \frac{2}{3}$$

$$i_o(t) = \frac{2}{3}e^{-5t}A, \qquad t > 0$$

7.13 Use the differential equation approach to find $i(t)$ for $t > 0$ in the circuit in Fig. P7.13 and plot the response, including the time interval just prior to opening the switch.

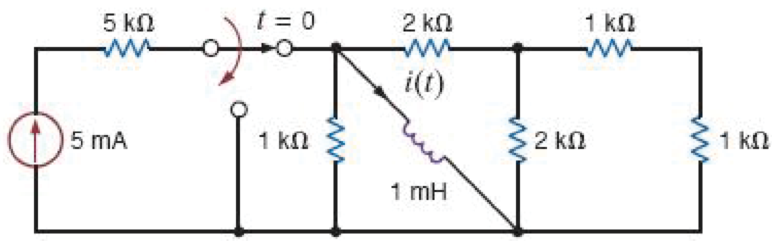
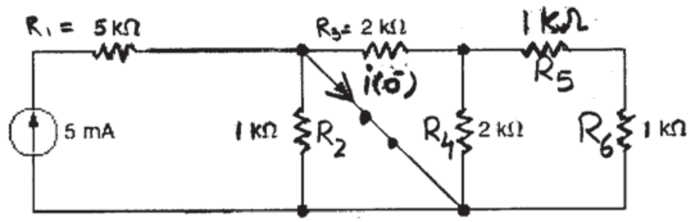


Figure P7.13

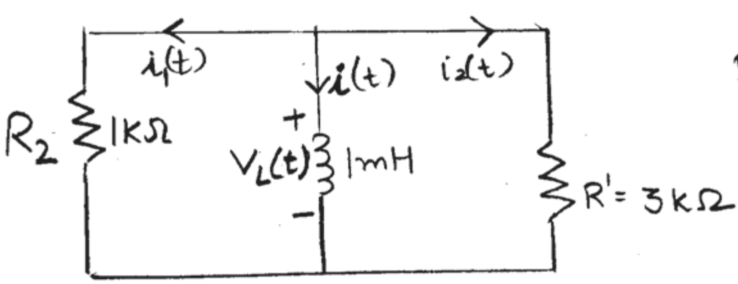
SOLUTION:

for $t = 0^-$



$i(0^-) = 5\,mA$, all of the current will flow through the short circuit.

for $t > 0$



$R' = [(R_5 + R_6) \| R_4] + R_3$

$R' = 3k\Omega$

KCL:

$$i_1(t)+i(t)+i_2(t)=0$$

$$\frac{V_L(t)}{R_2}+i(t)+\frac{V_L(t)}{R'}=0$$

$$V_L(t)=L\frac{di(t)}{dt}$$

$$\left(\frac{1}{R_2}+\frac{1}{R'}\right)L\frac{di(t)}{dt}+i(t)=0$$

$$\frac{di(t)}{dt}+\left(\frac{R_2R'}{R_2+R'}\right)\frac{i(t)}{L}=0$$

$$r+\frac{R_2R'}{(R_2+R')L}=0$$

$$r=\frac{-R_2R'}{(R_2+R')L}$$

$$i(t)=Ae^{rt}$$

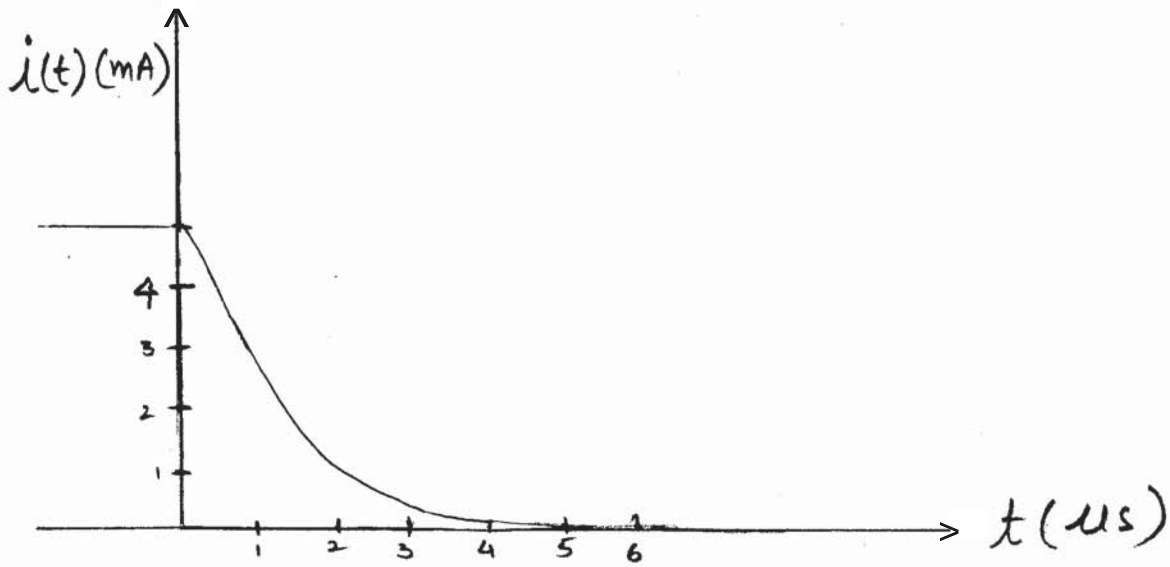
$$i(t)=Ae^{-\frac{R_2R'}{(R_2+R')L}t}$$

$$i(t)=Ae^{-7.5\times10^5t}$$

$$i(0^-)=5mA=i(0^+)$$

$$A=5\times10^{-3}$$

$$i(t)=5e^{-7.5\times10^5t}\ mA,\quad t>0$$


i(t) (mA)
4
3
2
1
1
2
3
4
5
6
t (μs)

7.20 Use the step-by-step method to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.20.

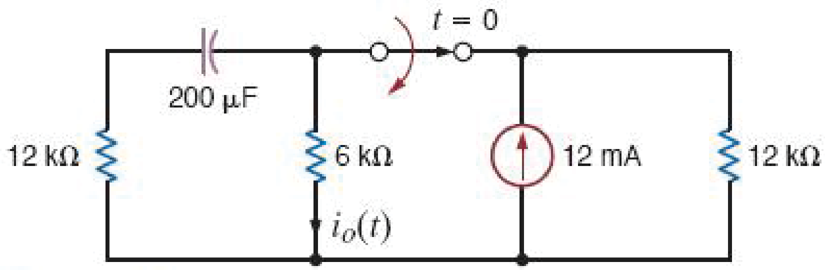
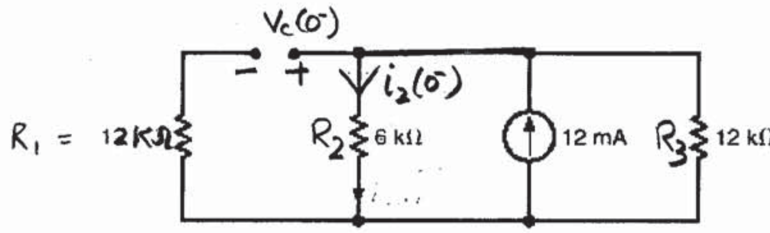


Figure P7.20

SOLUTION:

$t = 0^-$:



$$i_2(0^-) = \left(\frac{R_3}{R_2+R_3}\right)(12\times10^{-3})$$

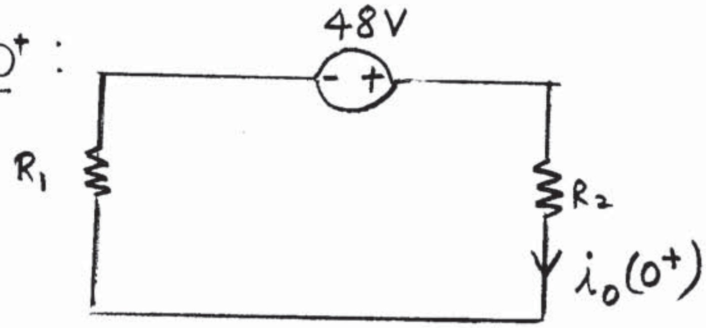
$$i_2(0^-) = 8\,mA$$

$$V_c(0^-) = i_2(0^-)(R_2)$$

$$V_c(0^-) = (8\times10^{-3})(6\times10^{3})$$

$$V_c(0^-) = 48V$$

$t = 0^+$:



$$i_o(0^+) = \frac{48}{R_1 + R_2}$$

$$i_o(0^+) = 2.67\text{mA}$$

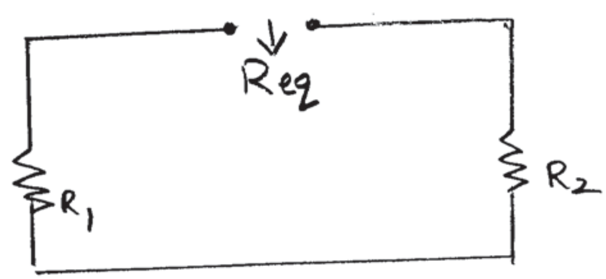
$$K_1 + K_2 = 2.67 \times 10^{-3}$$

$\underline{t = \infty:}$

$$i_o(\infty) = 0\text{A}$$

$$K_1 = 0$$

$$K_2 = 2.67 \times 10^{-3}$$



$$R_{eq} = R_1 + R_2$$

$$R_{eq} = 18\text{K}\Omega$$

$$\tau = R_{eq}C = (18\times10^{3})(200\times10^{-6})$$

$$\tau = 3.6\ s$$

$$i_o(t) = K_1 + k_2 e^{-t/\tau}$$

$$i_o(t) = 2.67\, e^{-t/3.6}\ mA$$

7.32 Use the step-by-step method to find $v_o(t)$ for $t > 0$ in the network in Fig. P7.32.

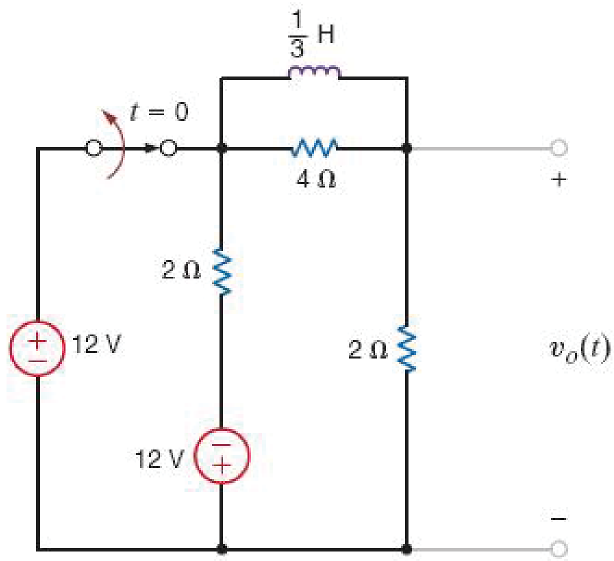
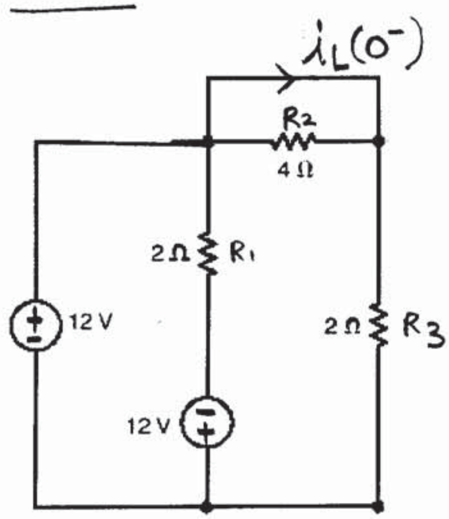


Figure P7.32

SOLUTION:

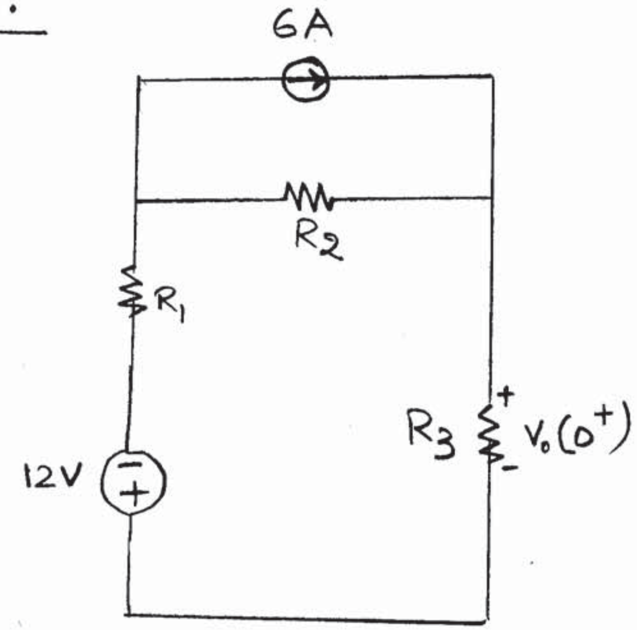
$t = 0^-$:

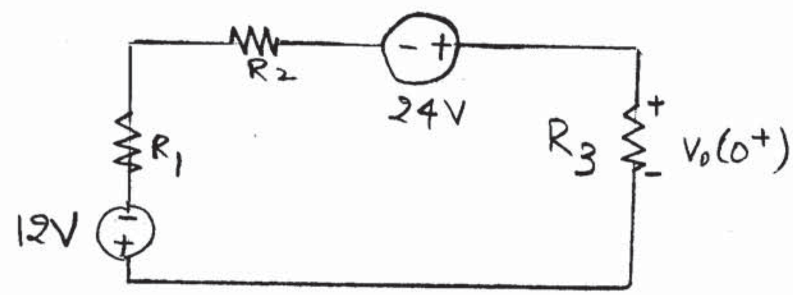


$$i_L(0^-) = \frac{12}{R_3} = \frac{12}{2}$$

$$i_L(0^-) = 6A$$

$\underline{t = 0^+:}$



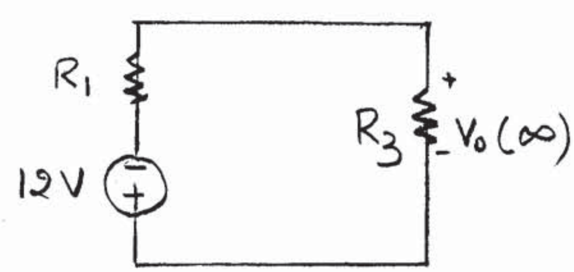


$$V_o(0^+) = \left(\frac{R_3}{R_1 + R_2 + R_3}\right)(24 - 12)$$

$V_o(0^+) = 3\text{V}$

$K_1 + K_2 = 3$

$\underline{t = \infty:}$



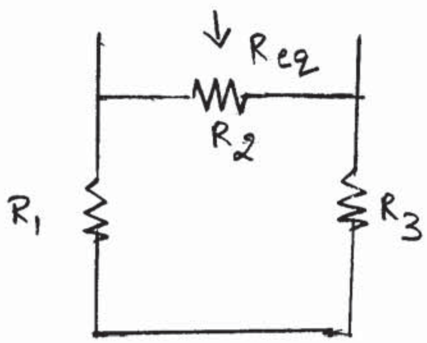
$$V_o(\infty) = \left(\frac{R_3}{R_1+R_3}\right)(-12)$$

$$V_o(\infty) = -6V$$

$$K_1 = -6$$

$$-6 + K_2 = 3$$

$$K_2 = 9$$



$$R_{eq} = R_2 \parallel (R_1 + R_3)$$

$$R_{eq} = 2\,\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{1/3}{2} = \frac{1}{6}\ s$$

$$V_o(t) = K_1 + K_2 e^{-t/\tau}$$

$$V_o(t) = -6 + 9e^{-6t}\ V, \quad t > 0$$

7.42 The switch in the circuit in Fig. P7.42 has been closed for a long time and is opened at $t = 0$. Find $i(t)$ for $t > 0$.

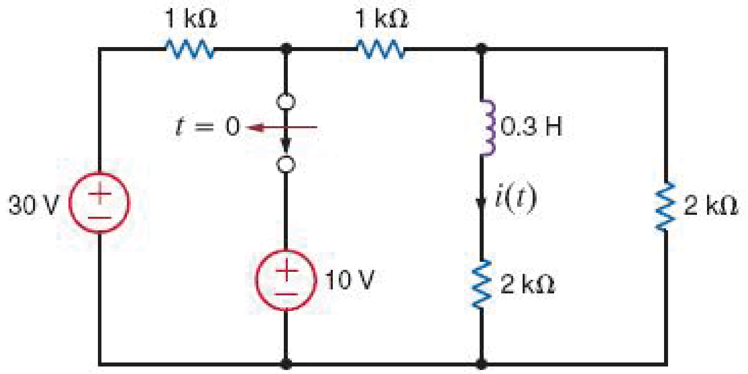
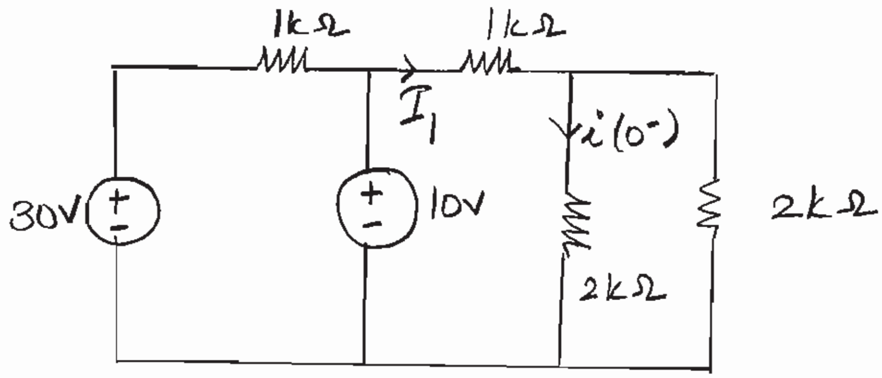


Figure P7.42

SOLUTION:

Step 1. $i(t) = K_1 + K_2 e^{-t/\tau}$

Step 2. $t = 0^-$



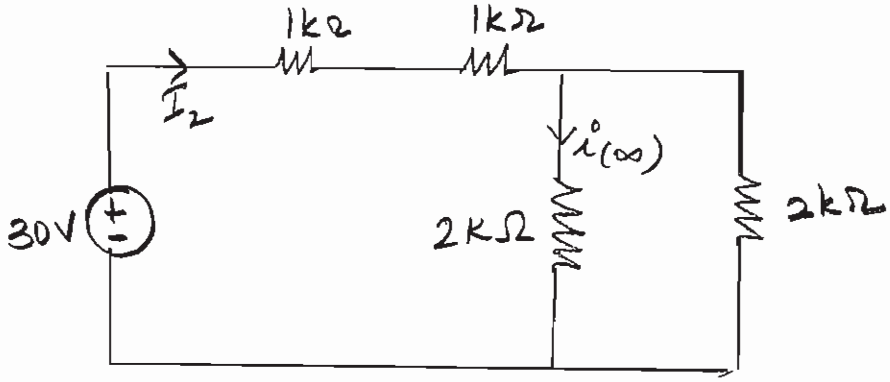
$$I_1 = \frac{10}{1k + \frac{(2k)(2k)}{2k + 2k}} = 5mA$$

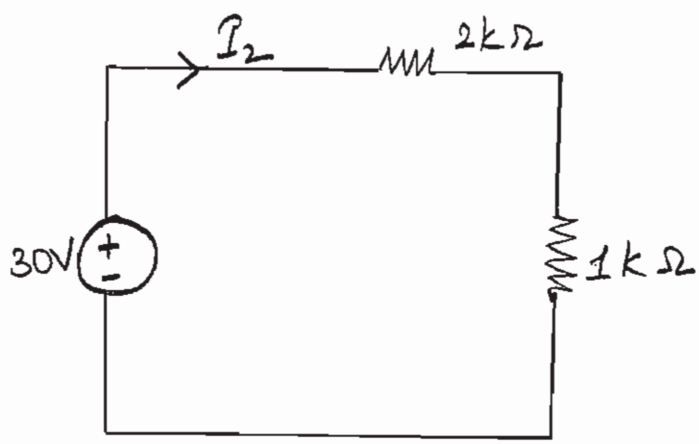
$$i(0^-) = 5m\left(\frac{2k}{2k + 2k}\right) = 2.5mA$$

Step 3. $t = 0^+$

$$i(0^-) = i(0^+) = 2.5mA$$

Step 4. $t = \infty$



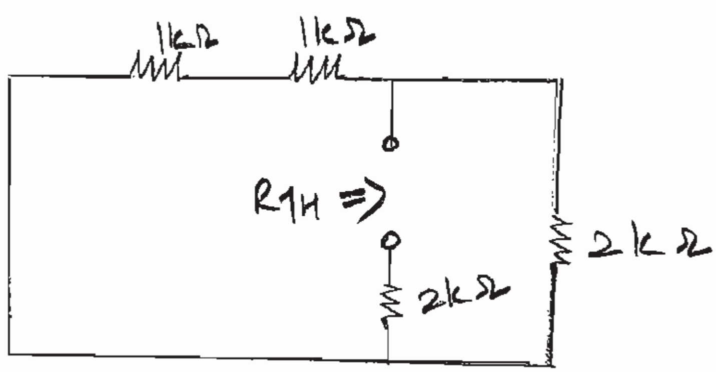


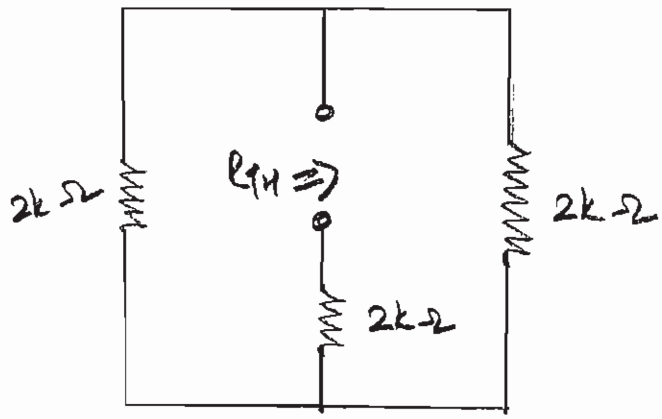
$$I_2 = \frac{30}{2k+1k} = 10mA$$

$$i(\infty) = 10m\left(\frac{2k}{2k+2k}\right)$$

$$= 5mA$$

Step 5. find τ





$$R_{TH} = 2k + \frac{(2k)(2k)}{2k + 2k}$$

$$R_{TH} = 3k\Omega$$

$$\tau = \frac{L}{R_{TH}} = \frac{0.3}{3 \times 10^3} = 10^{-4}\,s$$

Step 6. $i(t) = K_1 + K_2 e^{-t/\tau}$

$$i(\infty) = 5mA = K_1$$

$$i(0^+) = 2.5m = K_1 + K_2 \qquad K_2 = -2.5mA$$

$$i(t) = 5 - 2.5e^{-t/10^{-4}}\ mA, \quad t > 0$$