

University of Ottawa
MAT 1332 Practise Midterm
Duration: 80 Minutes. Instructor: Robert Smith?

Family Name: _____

First Name: _____

Do **not** write your student ID number on this front page. Please write your student ID number in the space provided on the second page.

Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- You have 80 minutes to complete this exam.
- This is a closed book exam, and no notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted**.
- Only the Faculty approved TI-30 calculator is allowed.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Where it is possible to check your work, do so.
- Good Luck!

Student number: _____, Total marks: _____ out of 30

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|---------|---|---|---|---|---|---|---|
| Problem | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Marks | | | | | | | |

Question 1. [4 points] Calculate

a) $\int_{-2}^2 \frac{1}{y^2 - 9} dy$ b) $\int_{-\pi/2}^{\pi/2} \frac{\sin(x)}{1 + \cos^2(x)} dx$

Question 2. [3 points] Solve the differential equation

$$\frac{dy}{dt} = \frac{6t \sin t}{y}$$

with initial condition $y(0) = 5$.

Question 3. [4 points] Evaluate the integral

$$\int \frac{x^3 + 2x^2 - 18x + 2}{x^2 + x - 12} dx.$$

Question 4. [6 points] For each of the following improper integrals, determine whether it converges, and determine its value if it does.

a) $\int_1^3 \frac{1}{t \ln t} dt$

b) $\int_0^\infty \frac{3}{18 + 2t^2} dt$

c) $\int_5^\infty \frac{\ln x}{x^2} dx$

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Question 5. [4 points] Define functions f and g by

$$f(x) = 3x^2 \qquad g = -3x + 6$$

- a) Show that f and g intersect at points $x = -2$ and $x = 1$.
- b) Calculate the area between f and g in the interval $-4 \leq x \leq 2$.

Question 6. [4 points] **a)** A very skinny 2m-long snake has density $\rho(x)$ kg/m at a distance x from the head of the snake, where

$$\rho(x) = 2x - x^2.$$

What is the total mass of the snake?

b) A less skinny 2m-long snake's body is given by rotating the function

$$y = \frac{e^{-x/2}}{10}, \quad 0 \leq x \leq 2$$

about the x -axis (the units of y is also m). What is the volume of this snake?

Question 7. [5 points] Determine the average value of $f(x) = x \ln(x)$ over the range $0 \leq x \leq 2$. (Hint: to calculate an indeterminate limit, rearrange and use L'Hôpital's rule.)