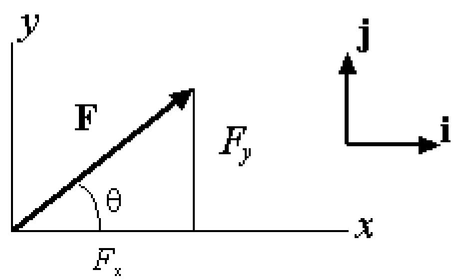
# Coordinates and Addition of Vectors - 2D

**Unit vector:** A vector of unit length

**Components of a vector in orthogonal bases:** Unit vectors **i** and **j** are along the *x* and *y* directions




$$\underline{\mathrm{F}} = F_x\,\underline{\mathrm{i}} + F_y\,\underline{\mathrm{j}}$$

$$F = \sqrt{F_x^2 + F_y^2}$$

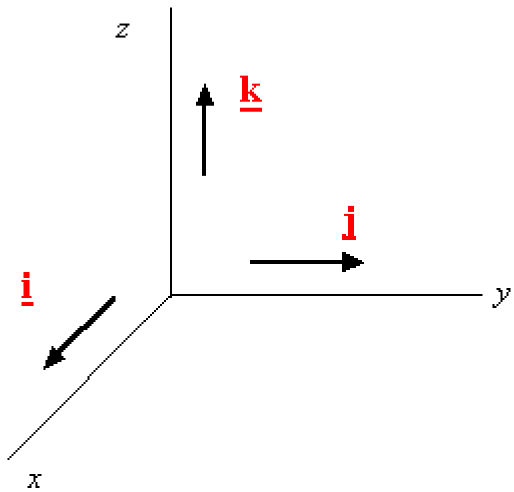
$$\tan(\theta) = \frac{F_y}{F_x}$$

**Addition of vectors using the components:**

$$\sum \underline{\mathrm{F}} = \sum F_x\,\underline{\mathrm{i}} + \sum F_y\,\underline{\mathrm{j}}$$

# Vectors in 3-D

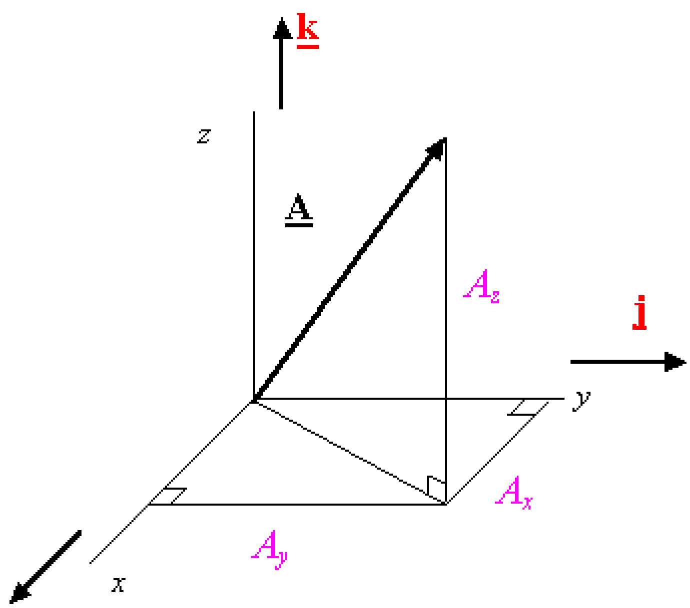
**Unit vector:** A vector of unit length.




**Base vectors for a rectangular coordinate system:** A set of three mutually orthogonal unit vectors

**Right handed system:** A coordinate system represented by base vectors which follow the right-hand rule.

**Rectangular component of a Vector:** The projections of vector **A** along the *x*, *y*, and *z* directions are $A_x$, $A_y$, and $A_z$, respectively.

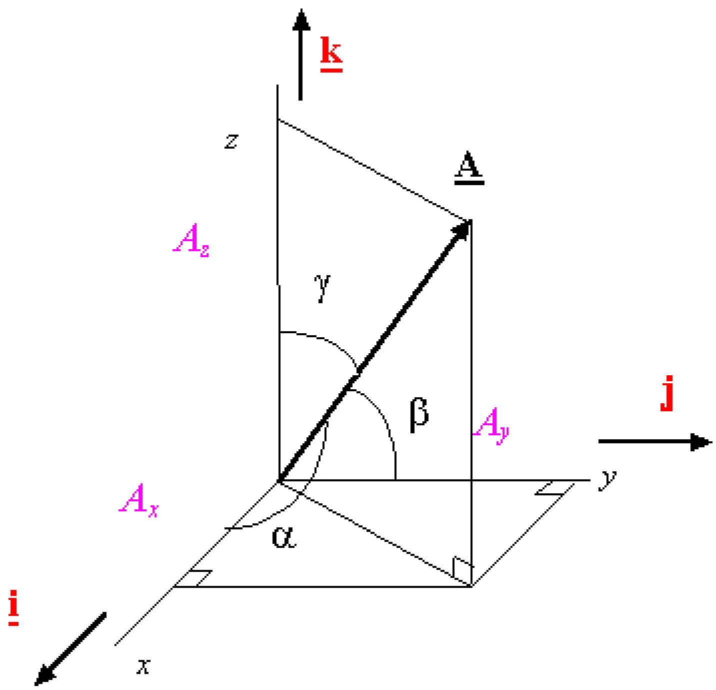


$$\underline{\mathbf{A}} = A_x\,\underline{\mathbf{i}} + A_y\,\underline{\mathbf{j}} + A_z\,\underline{\mathbf{k}}$$

**Magnitude of a Vector:**

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

**Direction Cosines: Cos(α), Cos(β), Cos(γ)**




$$\cos(\alpha) = \frac{A_x}{A}, \quad \cos(\beta) = \frac{A_y}{A}, \quad \cos(\gamma) = \frac{A_z}{A},$$

$$\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1$$

**Unit vector along a vector:** The unit vector $\underline{\mathbf{u}}_A$ along the vector $\underline{\mathbf{A}}$ is obtained from

$$\underline{\mathbf{u}}_A = \frac{\underline{\mathbf{A}}}{A} = \frac{A_x}{A}\underline{\mathbf{i}} + \frac{A_y}{A}\underline{\mathbf{j}} + \frac{A_z}{A}\underline{\mathbf{k}} = \cos(\alpha)\underline{\mathbf{i}} + \cos(\beta)\underline{\mathbf{j}} + \cos(\gamma)\underline{\mathbf{k}}$$

$$\underline{\mathbf{A}} = A\underline{\mathbf{u}}_A = A\cos(\alpha)\underline{\mathbf{i}} + A\cos(\beta)\underline{\mathbf{j}} + A\cos(\gamma)\underline{\mathbf{k}}$$

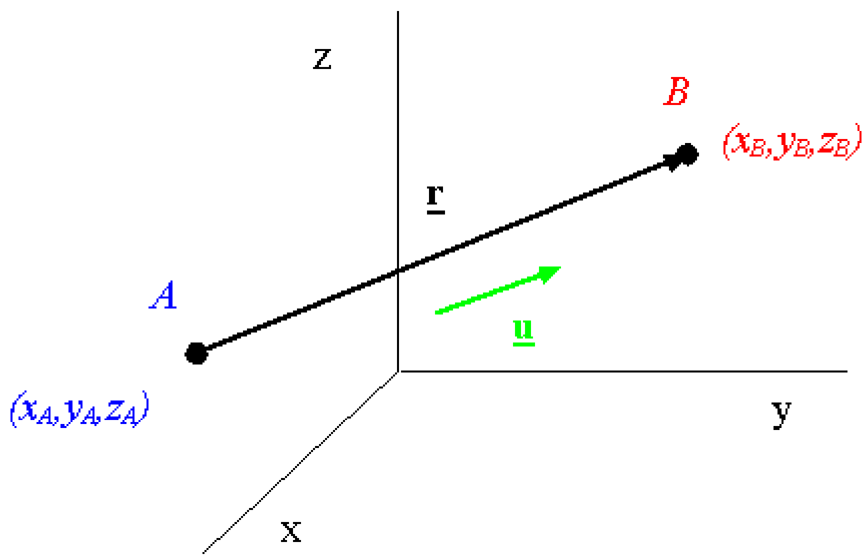
**Addition of vectors:** The resultant vector $\underline{\mathbf{F}}_R$ obtained from the addition of vectors $\underline{\mathbf{F}}_1$, $\underline{\mathbf{F}}_2$, …, $\underline{\mathbf{F}}_n$ is given by

$$\underline{\mathbf{F}}_R = \sum \underline{\mathbf{F}} = \sum F_x \underline{\mathbf{i}} + \sum F_y \underline{\mathbf{j}} + \sum F_z \underline{\mathbf{k}}$$

**Coordinates of points in space:** The triplet *(x,y,z)* describes the coordinates of a point.

**The vector connecting two points:** The vector connecting point *A* to point *B* is given by

$$\underline{\mathbf{r}} = (x_B - x_A)\underline{\mathbf{i}} + (y_B - y_A)\underline{\mathbf{j}} + (z_B - z_A)\underline{\mathbf{k}}$$



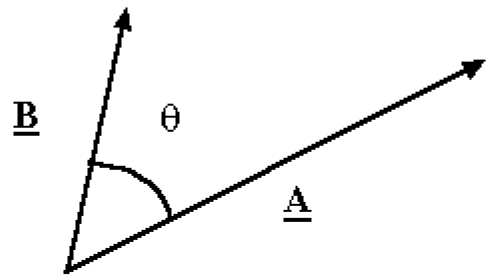

**A unit vector along the line *A*-*B*:** A unit vector along the line *A*-*B* is obtained from

$$\underline{\mathbf{u}} = \frac{\underline{\mathbf{r}}}{r}$$

**A vector along *A*-*B*:** A vector $\underline{\mathbf{F}}$ along the line *A*-*B* and of magnitude *F* can be obtained from

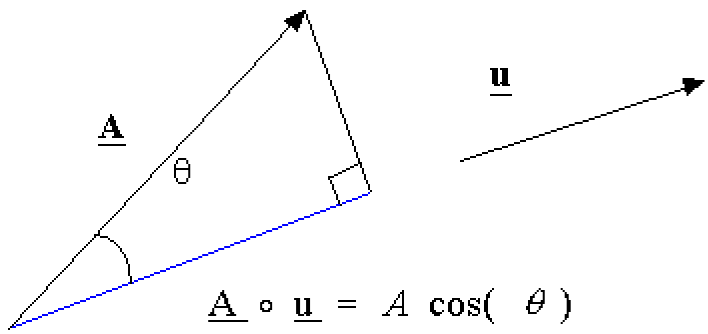
$$\underline{\mathbf{F}} = F\frac{\underline{\mathbf{r}}}{r} = F\underline{\mathbf{u}}$$

**The dot product:** The dot product of vectors $\underline{\mathbf{A}}$ and $\underline{\mathbf{B}}$ is given by




$$\underline{\mathbf{A}} \circ \underline{\mathbf{B}} = AB\cos(\theta) = A_x B_x + A_y B_y + A_z B_z$$

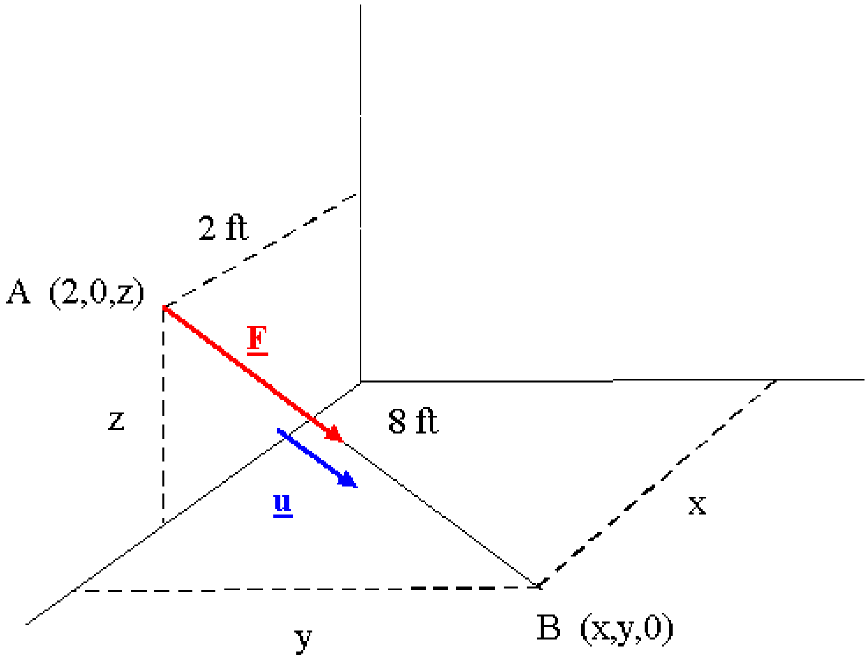
**Projection of a vector by using the dot product:** The projection of vector **A** along the unit vector **u** is given by




$$\underline{\mathbf{A}} \circ \underline{\mathbf{u}} = A\cos(\theta)$$

$$\textit{projection of } \underline{\mathbf{A}} \textit{ along } \underline{\mathbf{u}} = \underline{\mathbf{A}} \circ \underline{\mathbf{u}}$$

**Examples:**




- $$\begin{cases} \underline{\mathbf{r}} = (x_B - x_A)\underline{\mathbf{i}} + (y_B - y_A)\underline{\mathbf{j}} + (z_B - z_A)\underline{\mathbf{k}} \\ \underline{\mathbf{r}} = 8\underline{\mathbf{u}} = 8\dfrac{\underline{\mathbf{F}}}{F} \end{cases} \Rightarrow \begin{cases} \underline{\mathbf{r}} = (x-2)\underline{\mathbf{i}} + (y-0)\underline{\mathbf{j}} + (0-z)\underline{\mathbf{k}} \\ \underline{\mathbf{r}} = 8\dfrac{\underline{\mathbf{F}}}{F} = \dfrac{8}{17}(12\underline{\mathbf{i}} + 9\underline{\mathbf{j}} - 8\underline{\mathbf{k}}) \end{cases}$$

- $$\Rightarrow \begin{cases} x - 2 = \dfrac{(8)(12)}{17} \\ y = \dfrac{(8)(9)}{17} \\ -z = \dfrac{(8)(-8)}{17} \end{cases} \Rightarrow \begin{cases} x = 7.65 \quad ft \\ y = 4.24 \quad ft \\ x = 3.76 \quad ft \end{cases}$$

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