

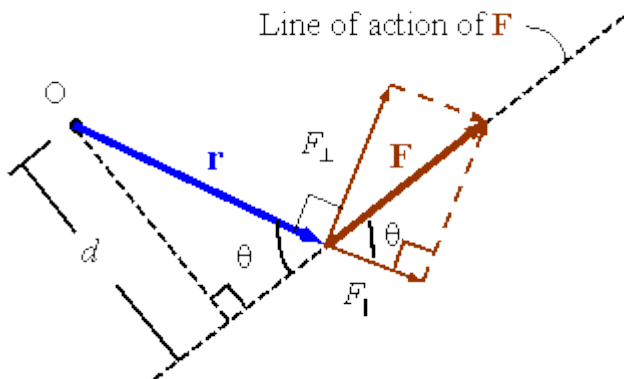
Moment of a force: Part 1

The magnitude of the moment of a force:

M_O : Magnitude of the moment of \mathbf{F} around point O

d : Perpendicular distance from O to the line of action of \mathbf{F}

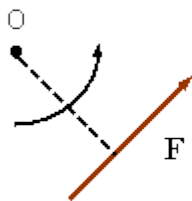
$$M_O = Fd$$



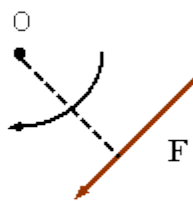
Note: moving a force along its line of action does not change its moment

Note: $M_O = Fd = Fr\sin(\theta) = rF_{\perp}$

Direction of the moment in 2-D: The direction of the moment is given by the right hand rule: Counter Clockwise (CCW) is out of the page, Clockwise (CW) is into the page.



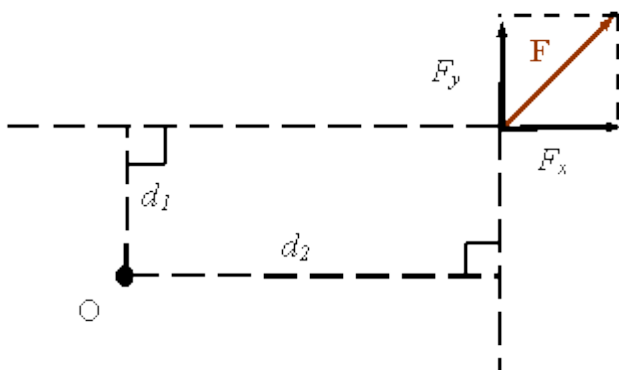
CCW-out of the page



CW-into the page

Calculating the moment in 2-D using components: Moments add together as vectors. Select a positive direction (CCW or CW), then calculate each moment and add them using the proper sign for each term. For example:

$$M_O = -F_x d_1 + F_y d_2 \quad \text{CCW positive}$$

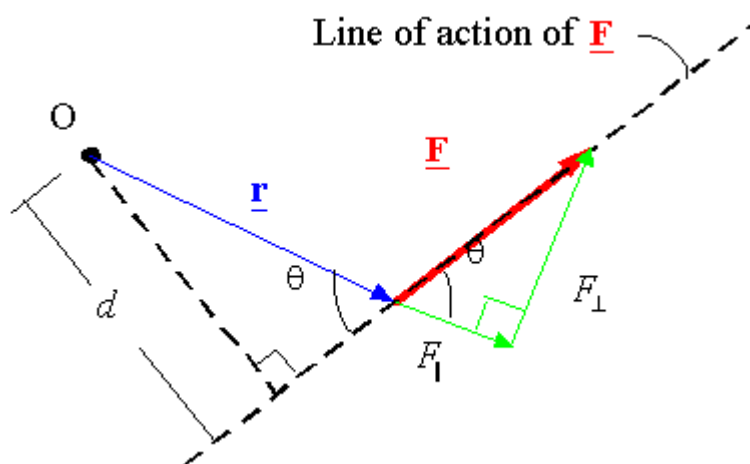


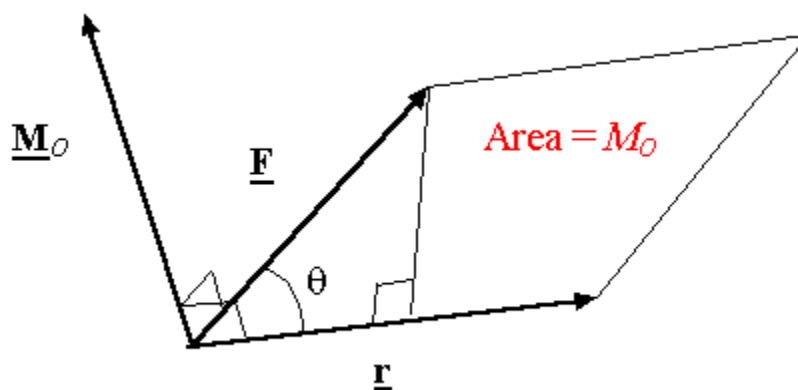
Moment of a force: Part 2

The cross product: [Math you need to know week 4-5](#)

Moment of Force $\underline{\mathbf{F}}$ around point \mathbf{O} : $\underline{\mathbf{M}}_O$

$$\underline{\mathbf{M}}_O = \underline{\mathbf{r}} \times \underline{\mathbf{F}}$$





$$M_O = |\mathbf{r} \times \mathbf{F}| = rF \sin(\theta) = Fd = rF_{\perp}$$

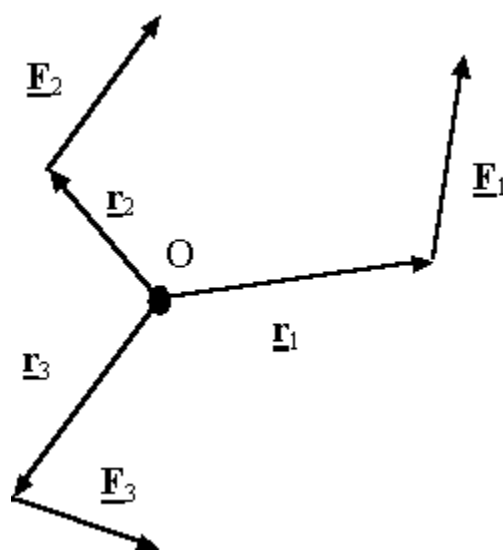
Calculating the moment using rectangular components:

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = (yF_z - zF_y)\mathbf{i} - (xF_z - zF_x)\mathbf{j} + (xF_y - yF_x)\mathbf{k}$$

Resultant moment: \mathbf{M}_O

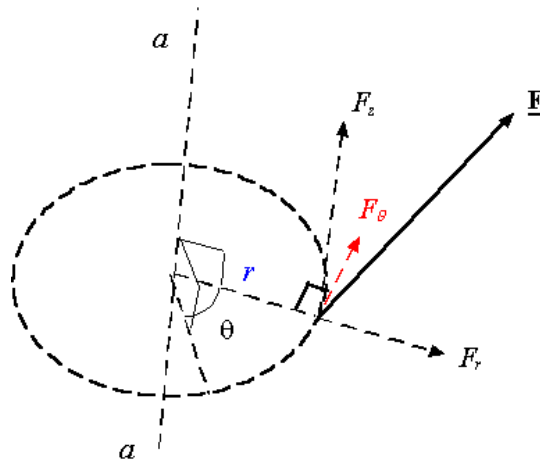
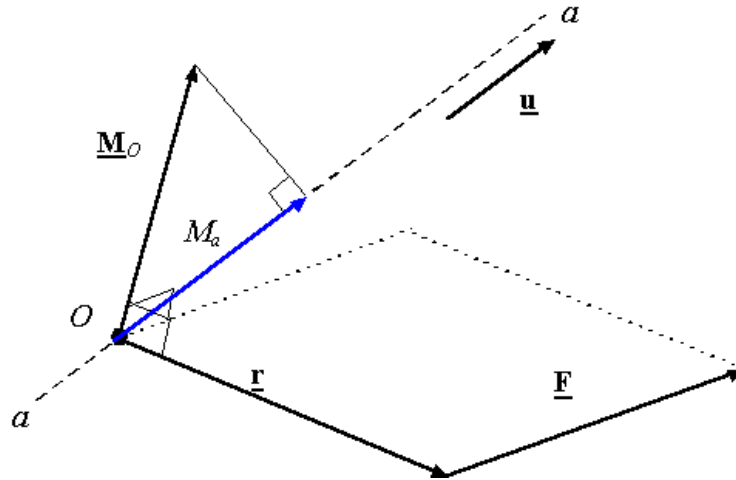


$$\underline{\mathbf{M}}_{R_O} = \underline{\mathbf{r}}_1 \times \underline{\mathbf{F}}_1 + \dots + \underline{\mathbf{r}}_n \times \underline{\mathbf{F}}_n = \sum \underline{\mathbf{r}} \times \underline{\mathbf{F}}$$

Moment of a force about a specified axis a - a : M_a

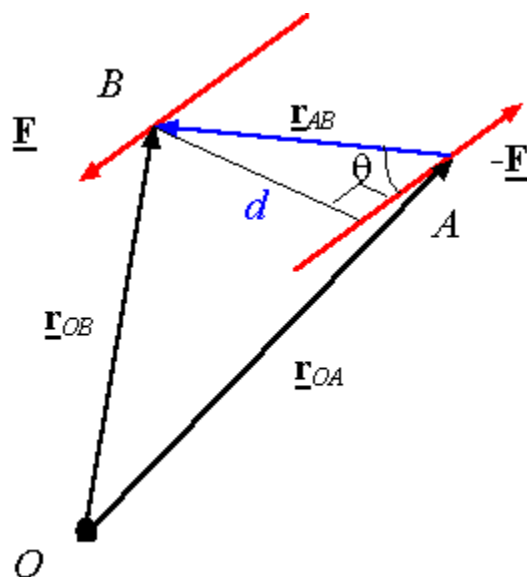
$$M_a = \underline{\mathbf{M}}_O \circ \underline{\mathbf{u}} = (\underline{\mathbf{r}} \times \underline{\mathbf{F}}) \circ \underline{\mathbf{u}}$$

O: any point on a - a



$$M_a = r F_\theta$$

Couple: \underline{C}



$$\underline{C} = \underline{M}_{R_O} = \underline{r}_{OB} \times \underline{F} + \underline{r}_{OA} \times (-\underline{F}) = (\underline{r}_{OB} - \underline{r}_{OA}) \times \underline{F} = \underline{r}_{AB} \times \underline{F}$$

$$C = r_{AB} F \sin(\theta) = Fd$$

Note: The moment of a couple does not depend on the point one takes the moment about. In other words, a moment of a couple is the same about all points in space.