ENGR 242

STATICS

Notes

Notation:

	Book	By hand
Scalar	a	a
Vector	**a (bold faced)**	$\underline{a}$ or $\vec{a}$

Units:

		SI		U.S.	
Mass	M	kilogram	kg	slug	
Length	L	meter	m	feet	ft
Force	F	Newton	N	pound	lb
Time	T	second	s	second	sec

Relation between units is based on the equation **F**=m**a:**

1 N = (1 kg) (1 m/s^2)
1 lb= (1 slug) (1 ft/sec^2)

Example of calculating mass in U.S. system: The mass, m, of a particle which weighs W=10 lb and is in a gravitational field of with an acceleration of gravity g=32.2 ft/sec^{2}is

$$m = \frac{W}{g} = \frac{10}{32.2}\,\text{slugs}$$

Unit conversion:

l lb	=	4.4482 N
1 slug	=	14.5938 kg
1 ft	=	0.3048 m
1 ft	=	12 in
1 mile	=	5,280 ft
1 kip	=	1,000 lb
1 ton	=	2,000 lb

Rounding numbers:

Round your final answers to 3 significant figures

$$1234 \rightarrow 1230 \quad or \quad 1.23 \times 10^{3}$$

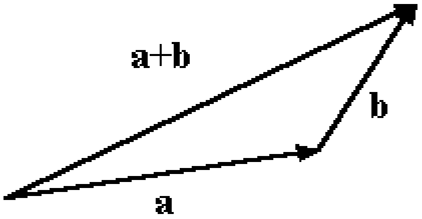
$$1235 \rightarrow 1240 \quad or \quad 1.24 \times 10^{3}$$

$$12351 \rightarrow 12400 \quad or \quad 1.24 \times 10^{4}$$

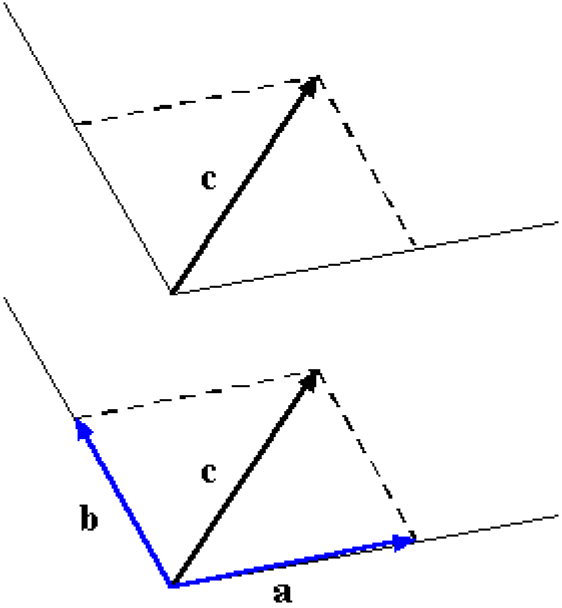
Check equations by checking that all terms in an equation have the same dimensions

Scalars and Vectors

- **Scalar:** A quantity like mass or temperature which only has a magnitude
- **Vector:** A quantity like heat flux or force which has both a magnitude and a direction (denoted by a bold faced character, an underlined character, or a character with a arrow on it)
- **Vector Addition:** Vector Addition follows the parallelogram law described be the figure

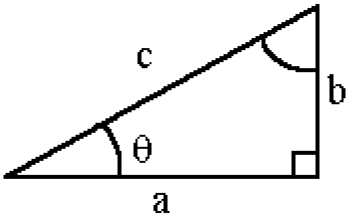


- **Resolution of a Vector:** A vector can be resolved along different directions using the parallelogram rule. The figure shows how one resolves vector $\underline{c}$ into components $\underline{a}$ and $\underline{b}$ which are along the given directions

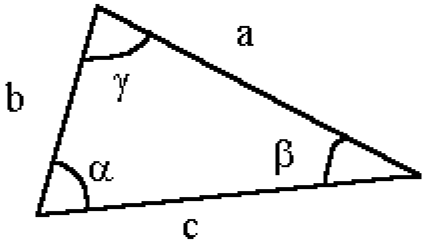


- **The math you need:**
    - **For a right triangle:**

$a^2 + b^2 = c^2$
tan(q) = b/a
sin(q) = b/c
cos(q) = a/c



For a general triangle:



alpha+beta+lamba=180^o

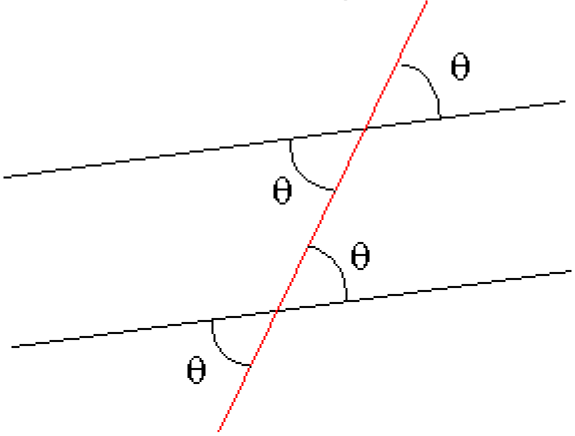
Sine law:

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

Cosine law:

$$c = \sqrt{a^2 + b^2 - 2ab\cos(\gamma)}$$

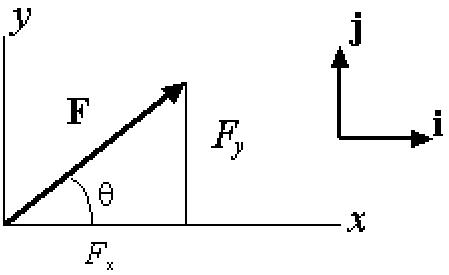
- **A line intersecting parallel lines:**



Coordinates and Addition of Vectors - 2D

Unit vector: A vector of unit length

Components of a vector in orthogonal bases: Unit vectors **i** and **j** are along the *x* and *y* directions



$$\underline{\mathrm{F}} = F_x\,\underline{\mathrm{i}} + F_y\,\underline{\mathrm{j}}$$

$$F = \sqrt{F_x^2 + F_y^2}$$

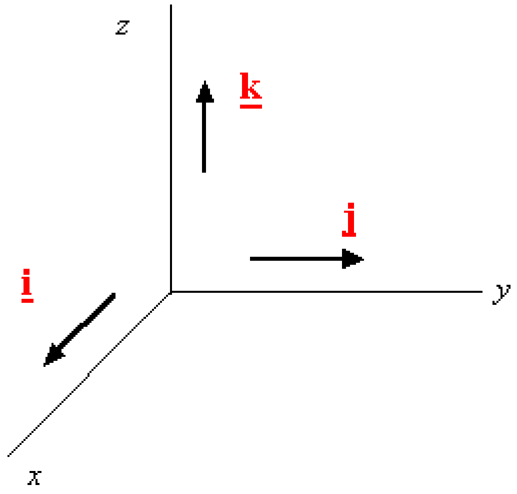
$$\tan(\theta) = \frac{F_y}{F_x}$$

Addition of vectors using the components:

$$\sum \underline{\mathrm{F}} = \sum F_x\,\underline{\mathrm{i}} + \sum F_y\,\underline{\mathrm{j}}$$

Vectors in 3-D

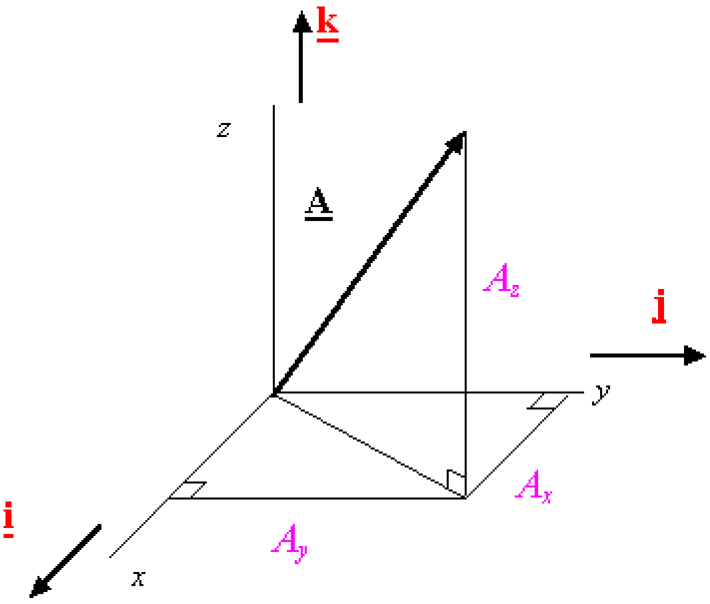
Unit vector: A vector of unit length.



Base vectors for a rectangular coordinate system: A set of three mutually orthogonal unit vectors

Right handed system: A coordinate system represented by base vectors which follow the right-hand rule.

Rectangular component of a Vector: The projections of vector $\underline{\mathbf{A}}$ along the x, y, and z directions are A_x, A_y, and A_z, respectively.

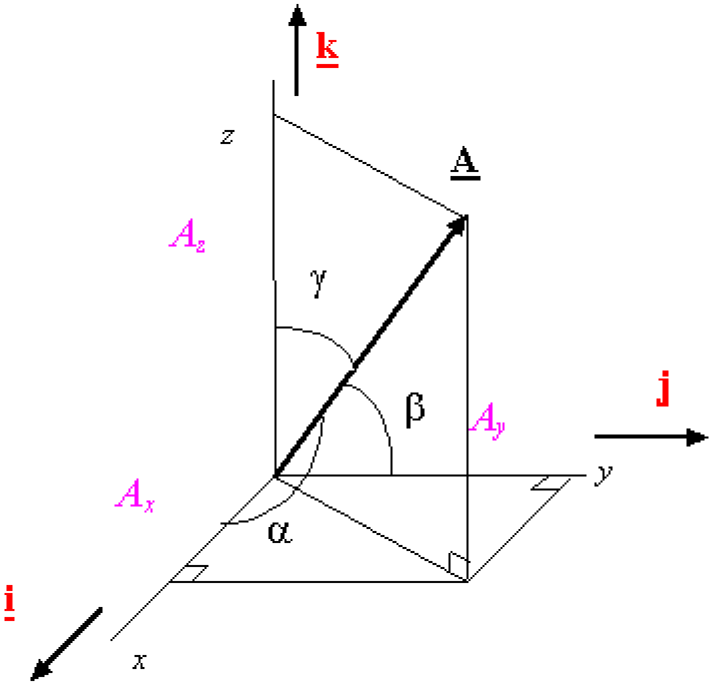


$$\underline{A} = A_x\,\underline{i} + A_y\,\underline{j} + A_z\,\underline{k}$$

Magnitude of a Vector:

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Direction Cosines: Cos(α),□Cos(β),□Cos(γ)



$$\cos(\alpha) = \frac{A_x}{A}, \quad \cos(\beta) = \frac{A_y}{A}, \quad \cos(\gamma) = \frac{A_z}{A},$$

$$\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1$$

Unit vector along a vector: The unit vector $\underline{u}_A$ along the vector $\underline{A}$ is obtained from

$$\underline{u}_A = \frac{\underline{A}}{A} = \frac{A_x}{A}\underline{i} + \frac{A_y}{A}\underline{j} + \frac{A_z}{A}\underline{k} = \cos(\alpha)\underline{i} + \cos(\beta)\underline{j} + \cos(\gamma)\underline{k}$$

$$\underline{A} = A\underline{u}_A = A\cos(\alpha)\underline{i} + A\cos(\beta)\underline{j} + A\cos(\gamma)\underline{k}$$

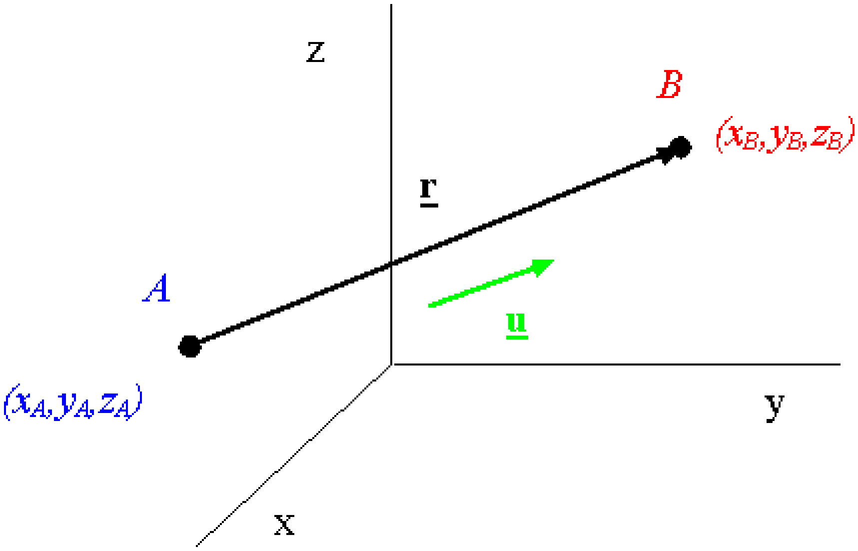
Addition of vectors: The resultant vector $\underline{\mathbf{F}}_R$ obtained from the addition of vectors $\underline{\mathbf{F}}_1$, $\underline{\mathbf{F}}_2$, …, $\underline{\mathbf{F}}_n$ is given by

$$\underline{\mathbf{F}}_R = \sum \underline{\mathbf{F}} = \sum F_x \underline{\mathbf{i}} + \sum F_y \underline{\mathbf{j}} + \sum F_z \underline{\mathbf{k}}$$

Coordinates of points in space: The triplet *(x,y,z)* describes the coordinates of a point.

The vector connecting two points: The vector connecting point *A* to point *B* is given by

$$\underline{\mathbf{r}} = (x_B - x_A)\underline{\mathbf{i}} + (y_B - y_A)\underline{\mathbf{j}} + (z_B - z_A)\underline{\mathbf{k}}$$



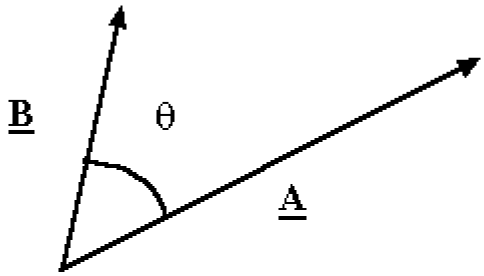
A unit vector along the line *A-B*: A unit vector along the line *A-B* is obtained from

$$\underline{\mathbf{u}} = \frac{\underline{\mathbf{r}}}{r}$$

A vector along *A-B*: A vector $\underline{\mathbf{F}}$ along the line *A-B* and of magnitude *F* can be obtained from

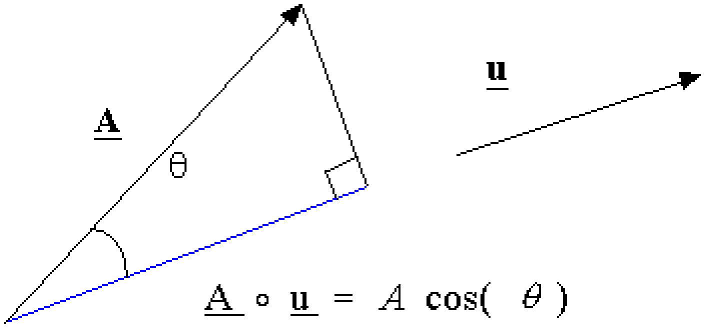
$$\underline{\mathbf{F}} = F\frac{\underline{\mathbf{r}}}{r} = F\underline{\mathbf{u}}$$

The dot product: The dot product of vectors $\underline{\mathbf{A}}$ and $\underline{\mathbf{B}}$ is given by



$$\underline{\mathbf{A}} \circ \underline{\mathbf{B}} = AB\cos(\theta) = A_x B_x + A_y B_y + A_z B_z$$

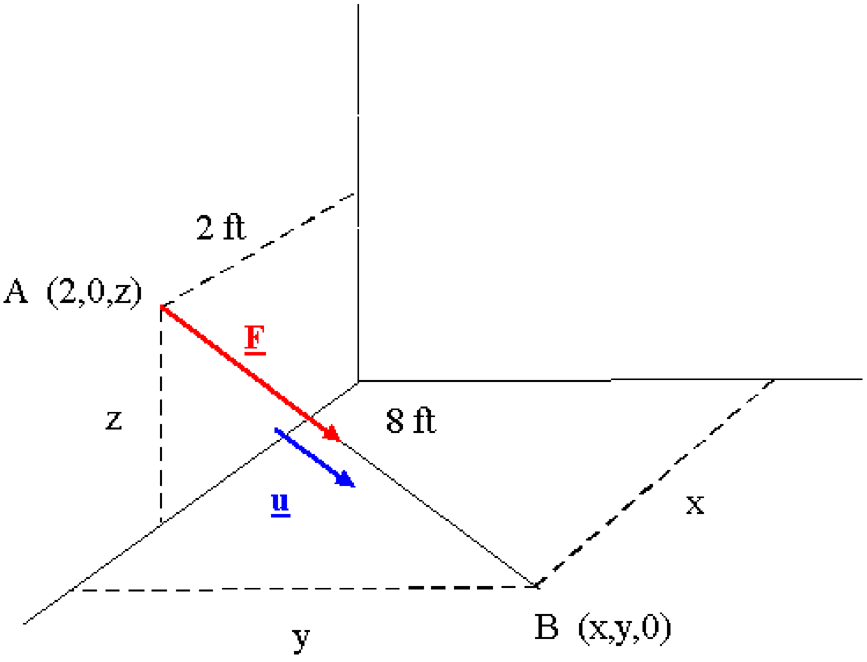
Projection of a vector by using the dot product: The projection of vector **A** along the unit vector **u** is given by



$$\underline{\mathbf{A}} \circ \underline{\mathbf{u}} = A\cos(\theta)$$

$$projection\,of\ \underline{\mathbf{A}}\ along\ \underline{\mathbf{u}} = \underline{\mathbf{A}} \circ \underline{\mathbf{u}}$$

Examples:



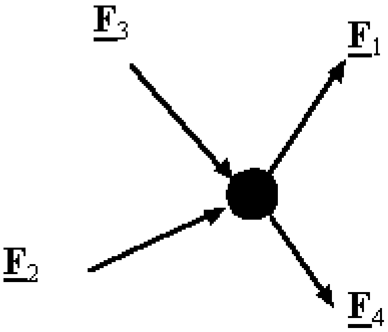
$$\begin{cases} \underline{\mathbf{r}} = (x_B - x_A)\underline{\mathbf{i}} + (y_B - y_A)\underline{\mathbf{j}} + (z_B - z_A)\underline{\mathbf{k}} \\ \underline{\mathbf{r}} = 8\underline{\mathbf{u}} = 8\dfrac{\underline{\underline{\mathbf{F}}}}{F} \end{cases} \Rightarrow \begin{cases} \underline{\mathbf{r}} = (x-2)\underline{\mathbf{i}} + (y-0)\underline{\mathbf{j}} + (0-z)\underline{\mathbf{k}} \\ \underline{\mathbf{r}} = 8\dfrac{\underline{\underline{\mathbf{F}}}}{F} = \dfrac{8}{17}(12\underline{\mathbf{i}} + 9\underline{\mathbf{j}} - 8\underline{\mathbf{k}}) \end{cases}$$

$$\Rightarrow \begin{cases} x - 2 = \dfrac{(8)(12)}{17} \\ y = \dfrac{(8)(9)}{17} \\ -z = \dfrac{(8)(-8)}{17} \end{cases} \Rightarrow \begin{cases} x = 7.65 \quad ft \\ y = 4.24 \quad ft \\ x = 3.76 \quad ft \end{cases}$$

Static Equilibrium for a Particle

A particle: An object with inertia (mass) but of negligible dimensions

Equilibrium equations for a particle: A particle is in equilibrium if the resultant of ALL forces acting on the particle is equal to zero



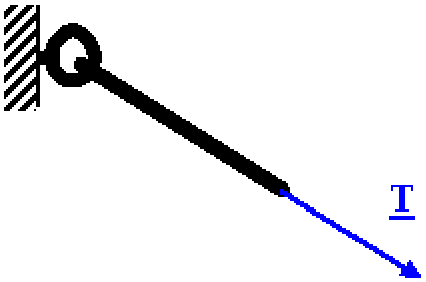
$$\sum \underline{\mathbf{F}} = 0$$

Equilibrium equations in component form: In a rectangular coordinate system the equilibrium equations can be represented by three scalar equations:

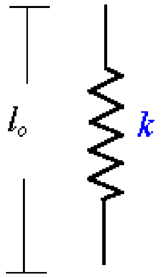
$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum F_z = 0$$

Free-Body diagram: A diagram showing the particle under consideration and all the forces acting on this particle. Each force in this diagram must labeled.

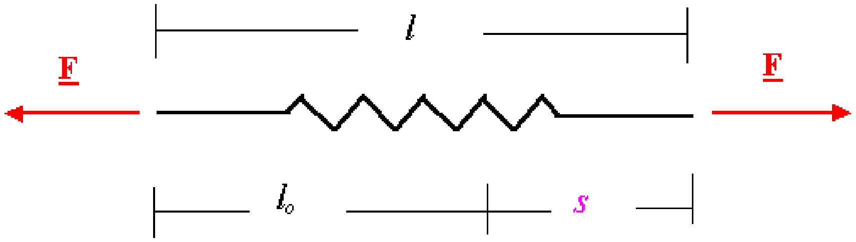
String or cable: A mechanical device that can only transmit a tensile force along itself.



Linear spring: A mechanical device which exerts a force along its line of action and proportional to its extension.

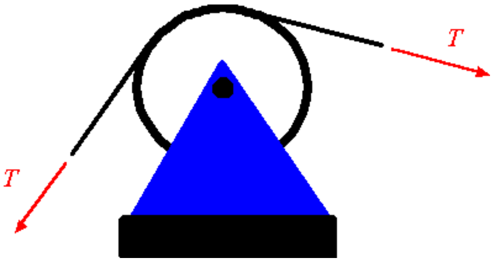


Free Spring



Stretched Spring

Frictionless pulleys: For a frictionless pulley in static equilibrium, the tension in the cable is the same on both sides of the pulley.

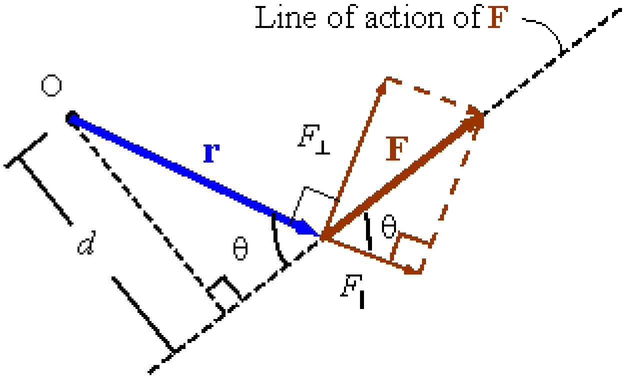


Moment of a force: Part 1

The magnitude of the moment of a force:

M_O: Magnitude of the moment of **F** around point O
d: Perpendicular distance from O to the line of action of **F**

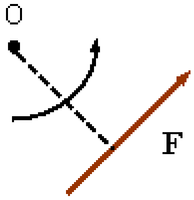
$$\boxed{M_O = Fd}$$



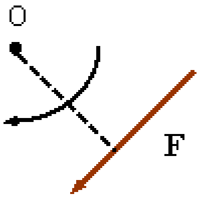
Note: moving a force along its line of action does not change its moment

Note: $M_0 = Fd = Fr\mathrm{Sin}(\theta) = rF_\perp$

Direction of the moment in 2-D: The direction of the moment is given by the right hand rule: Counter Clockwise (CCW) is out of the page, Clockwise (CW) is into the page.



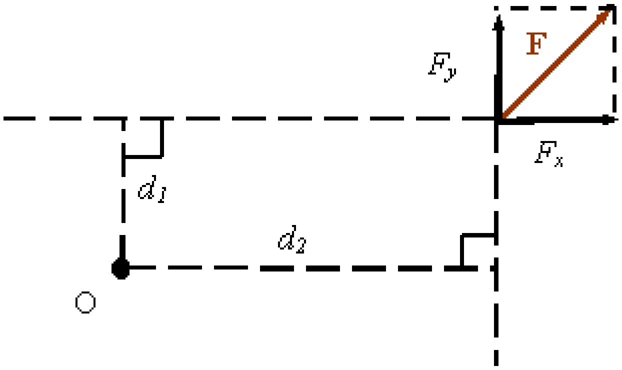
CCW-out of the page



CW-into the page

Calculating the moment in 2-D using components: Moments add together as vectors. Select a positive direction (CCW or CW), then calculate each moment and add them using the proper sign for each term. For example:

$$M_O = -F_x d_1 + F_y d_2 \quad CCW\ positive$$

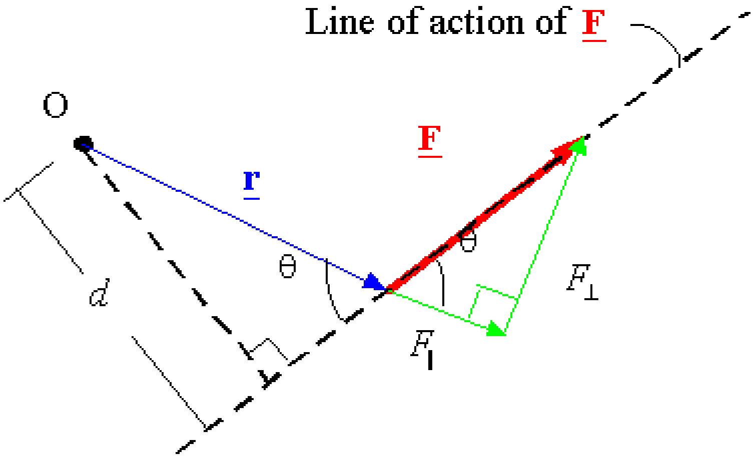


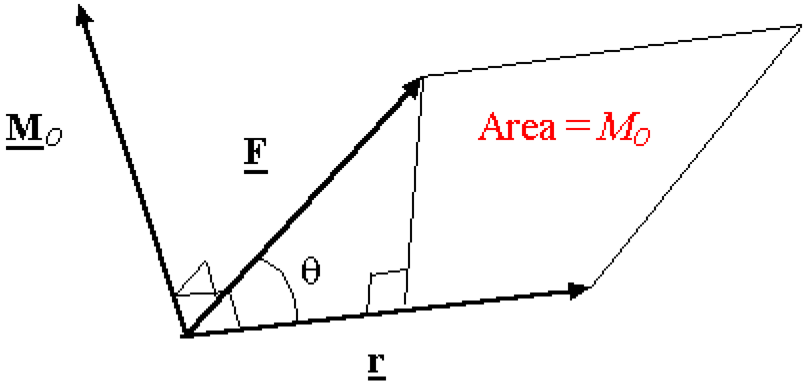
Moment of a force: Part 2

The cross product: Math you need to know week 4-5

Moment of Force $\underline{\mathbf{F}}$ around point O: $\underline{\mathbf{M}}_O$

$$\underline{\mathbf{M}}_O = \underline{\mathbf{r}} \times \underline{\mathbf{F}}$$





$$M_O = |r \times F| = rF\sin(\theta) = Fd = rF_{\perp}$$

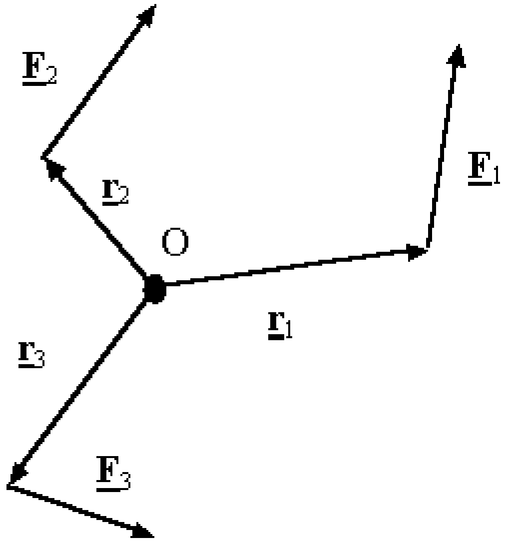
Calculating the moment using rectangular components:

$$\underline{\mathbf{r}} = x\underline{\mathbf{i}} + y\underline{\mathbf{j}} + z\underline{\mathbf{k}}$$

$$\underline{\mathbf{F}} = F_x\underline{\mathbf{i}} + F_y\underline{\mathbf{j}} + F_z\underline{\mathbf{k}}$$

$$\underline{\mathbf{M}}_O = \underline{\mathbf{r}} \times \underline{\mathbf{F}} = \begin{vmatrix} \underline{\mathbf{i}} & \underline{\mathbf{j}} & \underline{\mathbf{k}} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = (yF_z - zF_y)\underline{\mathbf{i}} - (xF_z - zF_x)\underline{\mathbf{j}} + (xF_y - yF_x)\underline{\mathbf{k}}$$

Resultant moment: $\underline{\mathbf{M}}_{ro}$

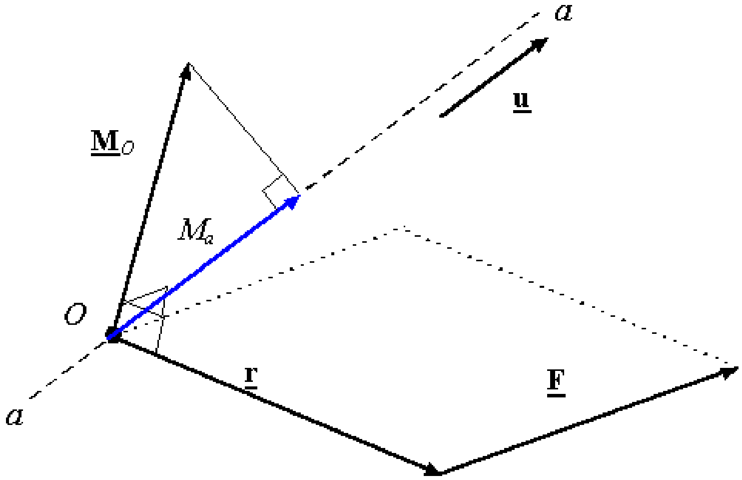


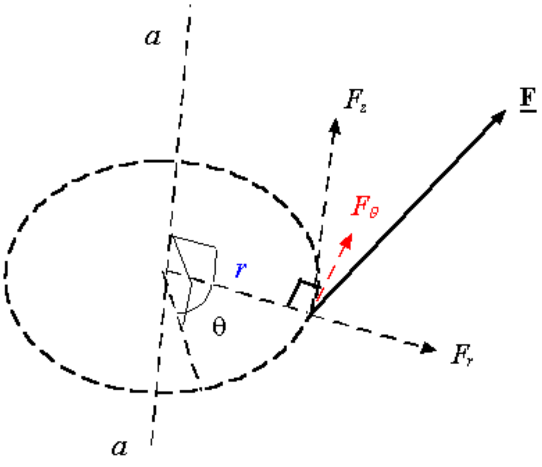
$$\underline{\mathbf{M}}_{R_O} = \underline{\mathbf{r}}_1 \times \underline{\mathbf{F}}_1 + \ldots + \underline{\mathbf{r}}_n \times \underline{\mathbf{F}}_n = \sum \underline{\mathbf{r}} \times \underline{\mathbf{F}}$$

Moment of a force about a specified axis *a-a*: M_a

$$M_a = \underline{\mathbf{M}}_O \circ \underline{\mathbf{u}} = (\underline{\mathbf{r}} \times \underline{\mathbf{F}}) \circ \underline{\mathbf{u}}$$

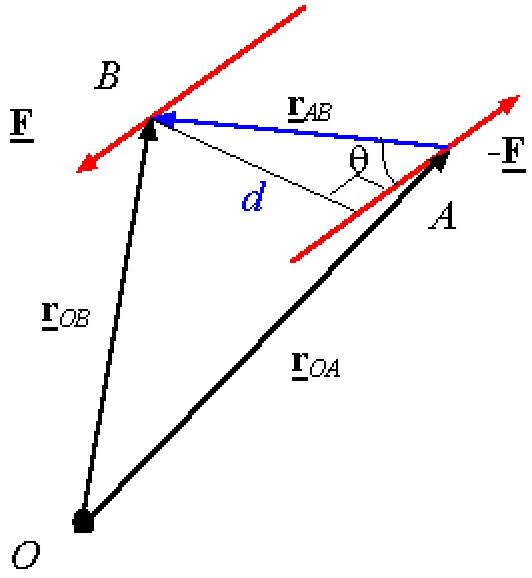
O: any point on *a*-*a*





$$M_a = rF_\theta$$

Couple: C



$$\underline{\mathbf{C}} = \underline{\mathbf{M}}_{R_O} = \underline{\mathbf{r}}_{OB} \times \underline{\mathbf{F}} + \underline{\mathbf{r}}_{OA} \times (-\underline{\mathbf{F}}) = (\underline{\mathbf{r}}_{OB} - \underline{\mathbf{r}}_{OA}) \times \underline{\mathbf{F}} = \underline{\mathbf{r}}_{AB} \times \underline{\mathbf{F}}$$

$$C = r_{AB} F \sin(\theta) = Fd$$

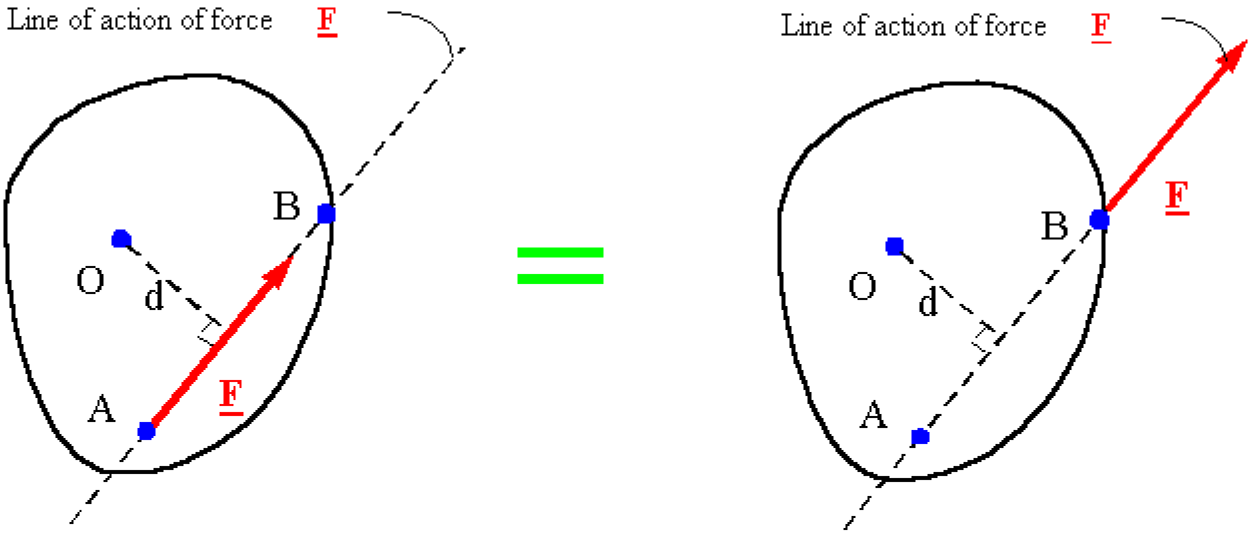
Note: The moment of a couple does not depend on the point one takes the moment about. In other words, a moment of a couple is the same about all points in space.

Equivalent force systems: Part 1

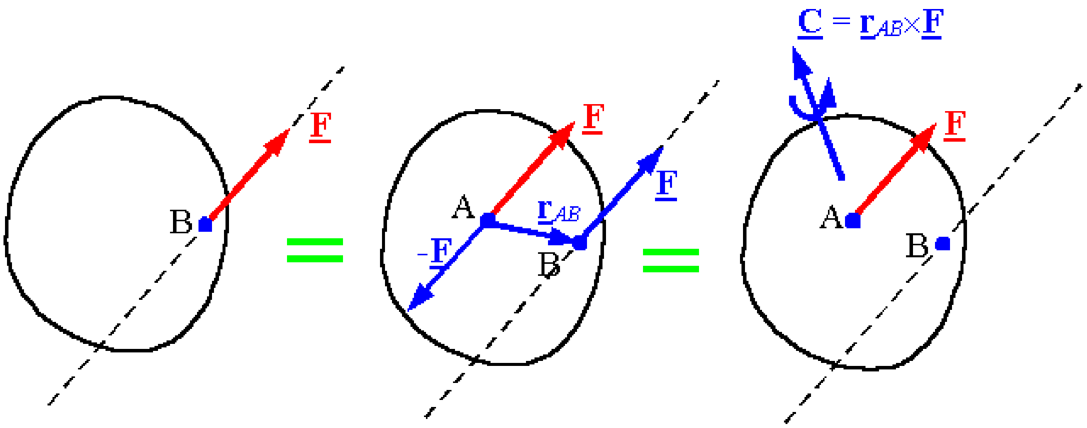
The basic idea: Two force systems are equivalent if they result in the same resultant force and the same resultant moment.

$$\begin{cases}\sum \underline{\mathbf{F}} \text{ for system } 1 = \sum \underline{\mathbf{F}} \text{ for system } 2 \\ \sum \underline{\mathbf{M}}_O \text{ for system } 1 = \sum \underline{\mathbf{M}}_O \text{ for system } 2\end{cases} \Leftrightarrow \text{The two force systems are equivalent}$$

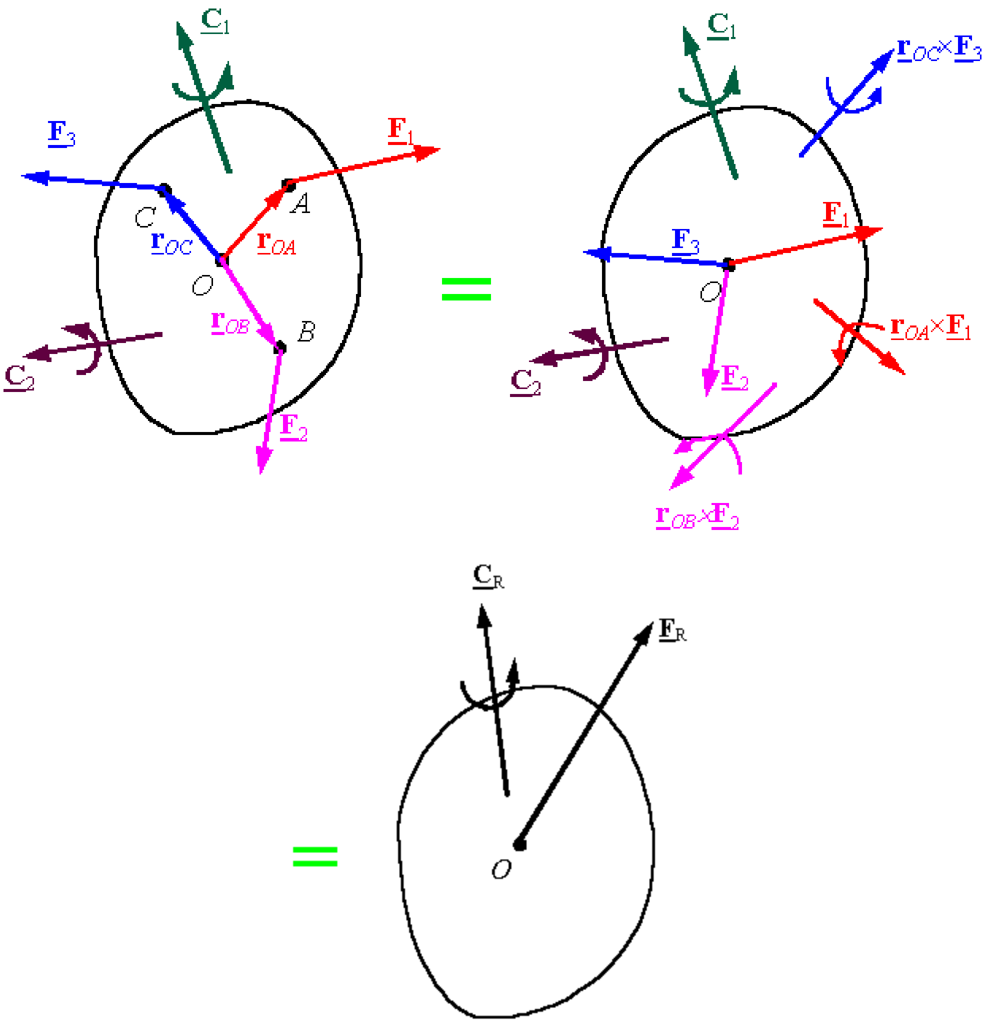
Moving a force along its line of action: Moving a force along its line of action results in a new force system which is equivalent to the original force system.



Moving a force off its line of action: If a force is moved off its line of action, a couple must be added to the force system so that the new system generates the same moment as the old system.



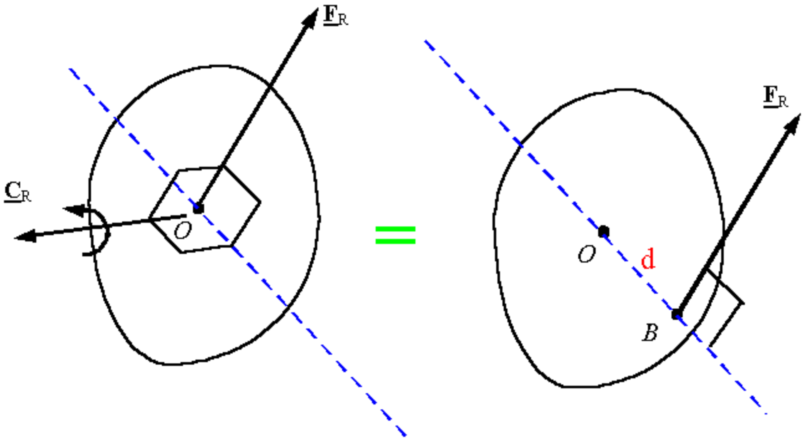
The resultant of a force and couple system: For any point *O*, every force and couple system can be made equivalent to a single force passing through *O* and a single couple. The single force passing through *O* is equal to the resultant force of the original system, and the couple is equal to the resultant moment of the original system around point *O*.



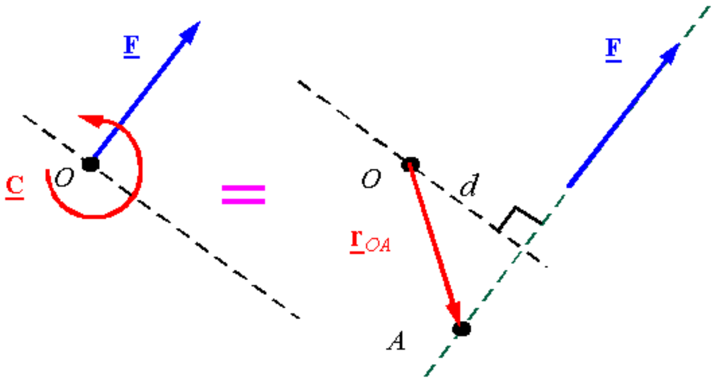
$$\begin{cases}\underline{\mathbf{F}}_R = \sum \underline{\mathbf{F}} \\ \underline{\mathbf{C}}_R = \sum \underline{\mathbf{M}}_O = \sum \underline{\mathbf{C}} + \sum \underline{\mathbf{r}} \times \underline{\mathbf{F}}\end{cases}$$

When can one reduce a force and couple system to a single force?: For a force and couple system if the resultant force and the resultant couple are perpendicular, then one can find an equivalent system with a single force and no couple. To obtain this system, move the resultant force a distance *d* along the line perpendicular to the plane of the resultant force and resultant couple until the resultant force creates a moment equivalent to the resultant couple.

$$F_R d = C_R \quad \Rightarrow \quad d = \frac{C_R}{F_R}$$



Note: All 2-D force systems can be reduced to a single force. To find the line of action of the force, the moment of the original system must be forced to be the same as the system with the single force.



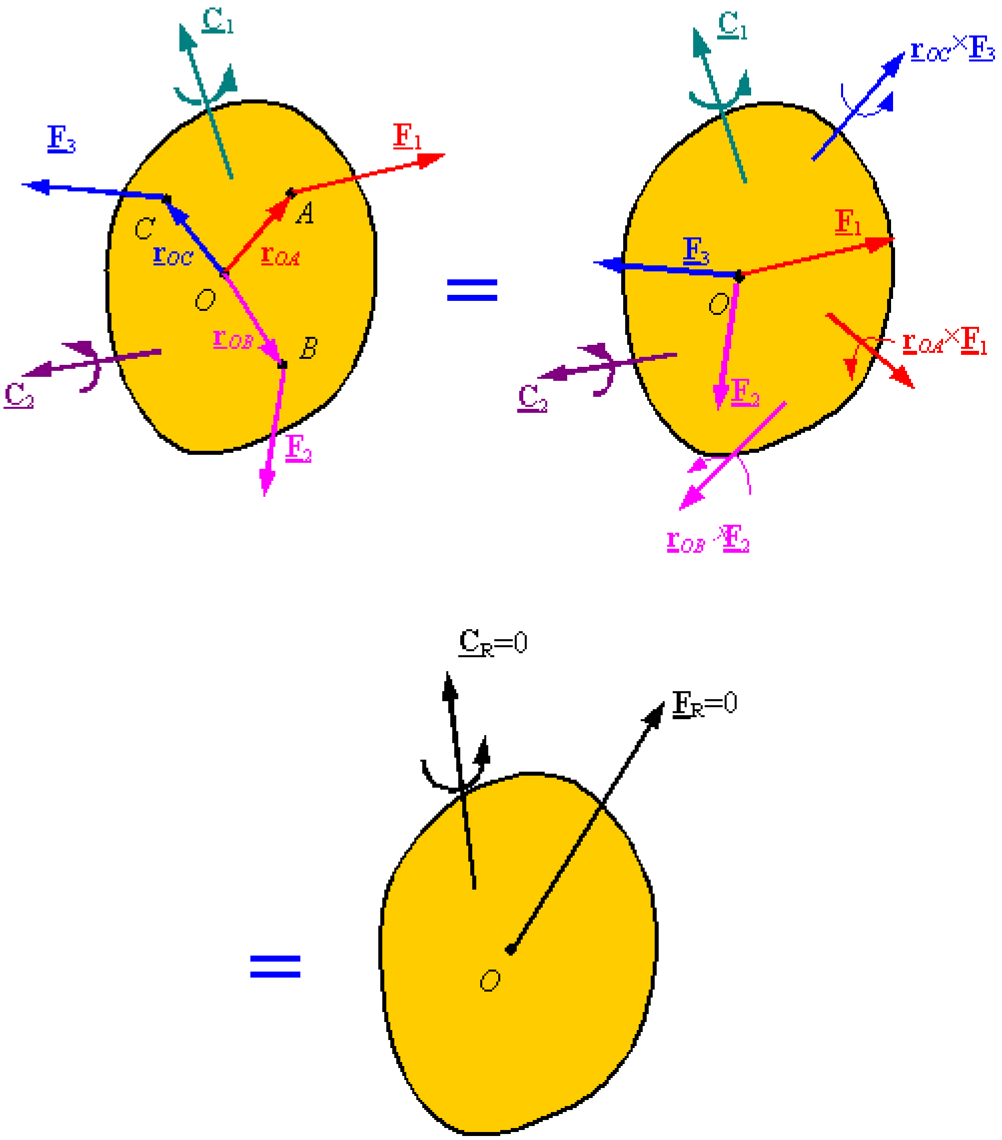
$$\underline{\mathbf{C}} = \underline{\mathbf{r}}_{OA} \times \underline{\mathbf{F}}$$

$$d = \frac{C}{F}$$

Equilibrium of rigid bodies

Static equilibrium for a rigid body: A body (or any part of it) which is currently stationary will remain stationary if the resultant force and resultant moment are zero for all the forces and couples applied on it.

$$\boxed{\begin{cases} \underline{\mathbf{F}}_R = \sum \underline{\mathbf{F}} = 0 \\ \underline{\mathbf{C}}_R = \sum \underline{\mathbf{M}}_O = \sum \underline{\mathbf{C}} + \sum \underline{\mathbf{r}} \times \underline{\mathbf{F}} = 0 \end{cases}}$$

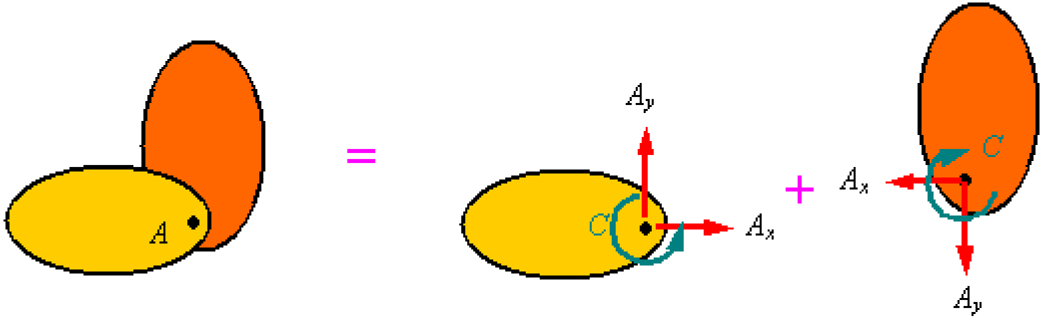


Newton's 3rd law: Each action has a reaction equal in magnitude and opposite in direction. This law provides the method used for one body (or part of a body) to interact with another body (or another part of the body).

Free-Body Diagram: A diagram of a body (or a part of it) which shows all the forces and couples applied on it, and which has all the forces and couples labeled for use in the solution of the problem is called a free-body diagram. Follow these steps to draw a free-body diagram.

1. Select the body (or part of a body) that you want to analyze, and draw it.
2. Identify all the forces and couples that are applied onto the body and draw them on the body. Place each force and couple at the point that it is applied.
3. Label all the forces and couples with unique labels for use during the solution process.
4. Add any relevant dimensions onto your picture.

Composite bodies and internal forces: Forces and couples which are a result of interaction between one part of an object and another part of it will not appear in the free-body diagram of the whole object. This is due to Newton's 3rd law. The two bodies in the following example are welded at A. When the two parts are looked at as a single body, the internal forces and couples are added together, and as a result of Newton's third law will cancel.

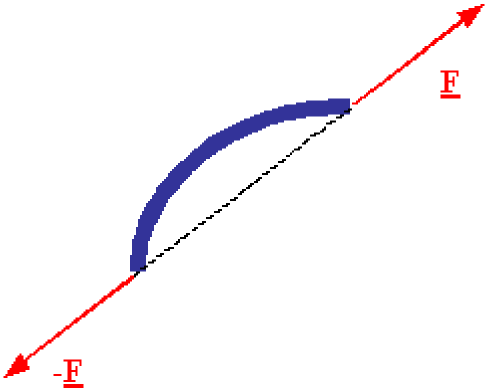


Forces and couples on a free-body diagram: Each force or couple you put on a free-body diagram represents a model of how the body in the free-body diagram is effected by its surroundings. In selecting the forces and couples that are to be applied on the free-body diagram follow these steps:

1. **Identify all the forces which come from the interaction of one body with another. Many of the common supports and their effects are shown in Table 5-1 on page 184. Remember that for each way in which a support restricts the free motion of the body, a force or a moment must be applied to the body to impose the restriction on the motion.**
2. **Apply the weight of the body to its center of gravity (if it is uniform, then apply it to the centroid).**
3. **Remember that strings and cables can only pull on an object.**
4. **Remember that internal loads cancel out and should not be put on the free-body diagram.**
5. **Remember that if you have selected the direction of forces or couples of interaction on one body, then Newton's 3rd law states that you must apply the forces or couples in the opposite direction on the other body.**

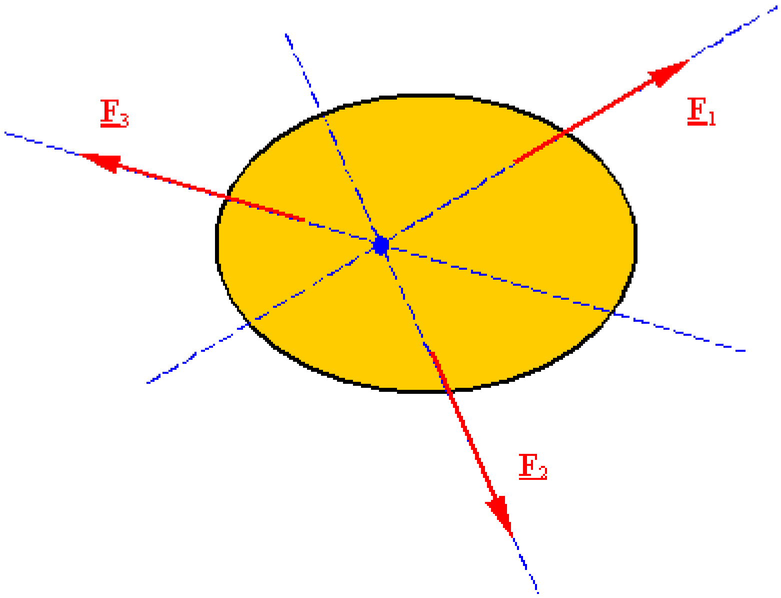
Solving for unknowns: You can write one set of equilibrium equations for each free-body diagram you draw. Things to remember are:

1. **In 2-D problems the equilibrium equations result in three independent scalar equations (two components of force and one component of moment). Therefore, you can only solve for three scalar unknowns.**
2. **Try to select the point you take moments around such that the line of action of at least one unknown force passes through that point. This will eliminate one unknown from your moment equation and will result in simpler equations to work with. This step is not essential, but will significantly simplify the algebra involved in solving your system of equations.**
3. **You can sometimes take moments about two or three different points in a problem. Select each point so that you eliminate one or more unknowns from the resulting moment equation. Remember that the additional equations you generate in this way are not independent of the original equations, and, therefore, you will still have only three independent equations in 2-D problems per free-body diagram and you can only solve for three unknowns per free-body diagram.**
4. **For a composite body, if you have drawn a free-body diagram and written the equilibrium equations for each of its subsections, you will gain no additional information if you draw the free-body diagram of the entire composite body and write its equilibrium equations.**
5. **In 3-D problems the equilibrium equations result in six independent scalar equations (three components of force and three components of moment). Therefore, you can solve for up to six scalar unknowns per free-body diagram.**



A two-force member: A body which has forces applied onto it at only two points, and no couples applied onto it at all, is called a two-force member. A two-force member can only be in equilibrium if the line of action of the resultant of the forces at each point passes through the other point, and each resultant force is equal in magnitude but opposite in direction to the resultant of the forces applied to the other point.

A three-force member: A body which has forces applied onto it at only three points, and no couples applied onto it at all, is called a three-force member. A three-force member can only be in equilibrium if the lines of action for the resultants of the forces at each point intersect at a single point.


$\underline{F}_3$
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