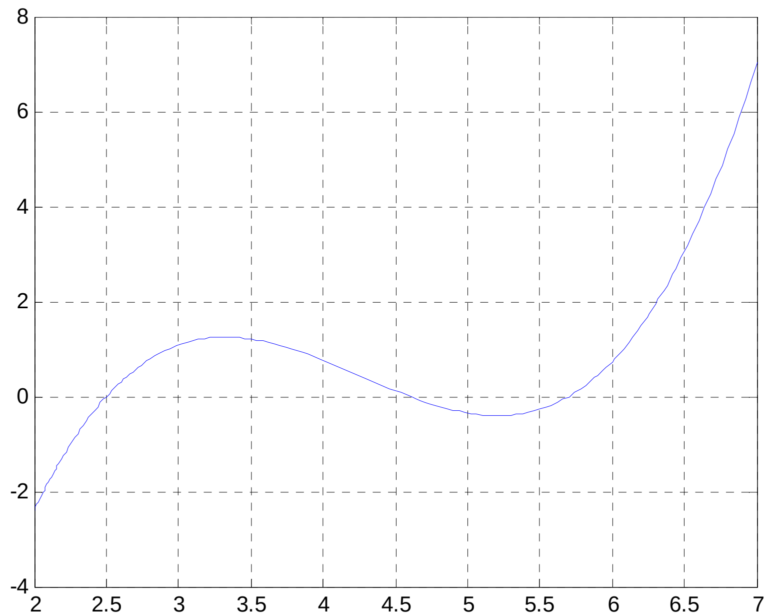
**** Section E Marking Scheme ****

Question 1 (6 marks)

The function plotted below has roots at $x = 2.5$, $x = 4.6$, and $x = 5.7$.



i) Suppose that a bisection search is started with $x_{LOW} = 5$ and $x_{HIGH} = 7$. Complete the following table.

Step	X_{LOW}	X_{HIGH}	X_{ROOT}	E_{MAX}
1	5	7	6	1
2	5	6	5.5	0.5
3	5.5	6	5.75	0.25

0.5 mark for consistently picking XROOT properly
1 mark for each correct wall movement (2 marks total)
0.5 mark for EMAX column

ii) How many ADDITIONAL steps would be required to reduce E_{MAX} to 0.0001?

total guesses required = 14.2877 (from formula with $\Delta x^0 = 2$, $E_{MAX} = 0.0001$)
can't have fractional guesses, therefore 15 total guesses required
12 additional steps required. check: $0.25/2^{13} = 0.00003 < 0.0001$
1 mark for the correct answer, 0.5 marks for a small slip

iii) Assuming that function *f* corresponds to the function shown above, what would happen if the following Matlab command were executed?

```
x = fzero (f, [4, 6]);
```

An error message ("function signs at end points must differ") will be output.
1 mark for indicating precisely what will happen (correct error message).
0.5 marks for just recognizing that an error will occur.

(iv) Would a bisection search started with $x_{LOW} = 2$ and $x_{HIGH} = 6$ find one of the roots and, if so, which of the roots would it find? You must provide enough of an explanation that it is clear that your answer is not just a guess.

It would find the root at 2.5. The initial guess would 4, the high wall would move down, and after this there would only be only root within the interval.
1 mark for giving both the root that will be found and an clear explanation
0.5 marks for giving the right root but not explaining things very well
no marks for the wrong root or no explanation at all

Question 2 (3 marks)

Assume the following system of three simultaneous equations.

$$a = 2(b + c) + 9$$
$$b = 4(a + b + c) - 5$$
$$c = 2(a - 3)$$

Give the Matlab code required to find and output the values of *a*, *b*, and *c*.

```
A = [ 1 -2 -2; -4 -3 -4; -2  0 1]; b = [9 -5 -6]';
x = A\b;
fprintf ('a = %f, b = %f, c = %f\n', x);
```

1 mark for getting the problem into Ax = b form (A and b correct)
0.5 marks for correctly defining matrices
1 mark for left division
0.5 marks for outputting results (just leaving off the semi-colon is not acceptable)

Question 3 (6 marks).

Suppose that we need to find the values of x that satisfy $x^2 - 2x = 4$.

i) Express the problem in root finding form and give the Newton-Raphson iterative formula for this particular problem.

.

root finding form: $x^2 - 2x - 4 = 0$ 1 mark

formula: $x_{i+1} = x_i - (x_i^2 - 2 x_i - 4) / (2 x_i - 2)$ 1 mark

ii) Assume a Newton Raphson search starting with $x_0 = 4$. Complete the following table.

i	x_i	E_A
0	4	N/A
1	3.3333333	.6666666
2	3.238095	0.0952383

Correct values in x_i column: 1 mark
Values in E_A correct are correct GIVEN THE X_I VALUES: 1 marks

iii) Give one reason why the Newton-Raphson approach might be preferred to a bisection search.

Faster. Does not require that we have lower and upper limits.
Award 0.5 marks for any reasonable answer.

iv) Give one reason why a bisection search might be preferred to a Newton-Raphson search.

Safer. Error in solution is guaranteed to be within some amount of the correct answer.
Award 0.5 marks for any reasonable answer.

v) Give Matlab code that uses function *roots* to finds both of the possible solutions and that outputs them in a message of the form "The two possible solutions are x = ... and x = ...".

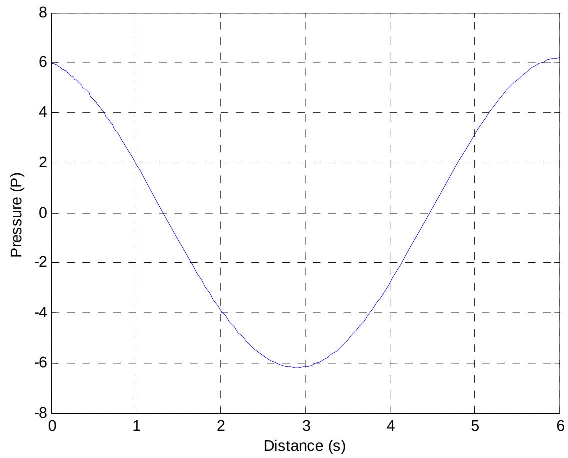
```
p = [ 1 -2 -4];
x = roots(p)
fprintf ('The two possible solutions are x = %f and x = %f\n', x);
```

1 mark for getting all of the above (including the output statement) correct
0.5 marks for defining the right polynomial and passing it to function roots.

Question 4 (5 marks)

The pressure behind an airfoil varies with the distance from its leading edge according to

$P(s) = 6\cos(s) - 1.5\sin(s)$ where P is the pressure and s is the distance



Note: The formula assumes that "sin" and "cos" work in radians.
Reminder: the derivative of *cos(x)* is *-sin(x)*, the derivate of *sin(x)* is *cos(x)*

We want to find the distance at which the pressure is lowest.

i) One possible approach is to convert the problem into a root finding problem. In the space below give Matlab commands that implement this approach and output the answer using *fprintf*.

```
fp = @(s) -6*sin(s) - 1.5*cos(s);
s = fzero (f, [2 4]);
fprintf ('The pressure is least at s = %f\n', s);
```

0.5 marks for the concept of taking the derivative
0.5 marks for defining the correct function
1 mark for using fzero with appropriate limits
0.5 marks for outputting the answer

ii) Matlab also allows the distance where the pressure is minimal to be found directly from the given function. In the space below give Matlab commands that implement this approach and output the answer using *fprintf*.

```
fp = @(s) 6*coss) - 1.5*sin(s);
s = fminbnd (f, 2, 4);
fprintf ('The pressure is least at s = %f\n', s);
```

1 mark for defining the correct function
1 mark for using fminbnd with appropriate limits
0.5 marks for outputting the answer

Question 5 (5 marks)

The following function has a root in the vicinity of 5.

$$f(x) = 4 - 2^{(x^2/10)}$$

A secant search starting with $x_0 = 2$ and $x_1 = 4$ will converge on this root. Demonstrate your understanding of this process by completing the following table.

i	$\mathbf{X_i}$	$\mathbf{f(X_i)}$	$\mathbf{E_A}$
0	2	2.6805	XXXX
1	4	0.9686	XXXX
2	5.131553	-2.204364	1.131553
3	4.345417	XXXX	0.786136

1 mark for getting X_2 right
1 mark for getting X_3 right as well
1 mark for getting the E_A column right GIVEN THE X_i VALUES

Suppose that $x_0 = 2$ (as above). Give a value for x_1 that will cause the search to fail and briefly explain why this will happen.

x_1 = ______________-2__________ Denominator of iterative formula becomes zero.

2 marks for the above answer
1 mark for choosing $x_1 = 2$ and giving the same reason (a bit of a trick answer)

Question 6 (5 marks)

Some function $f(x)$ has a minimum between $x = 4$ and $x = 12$. Assuming a Golden Section search on this interval, where will the two test points x_1 and x_2 initially be located?

x_1 = ______8.94427________ x_2 = _____7.05572______________

1 mark for each value

If $f(x_1) < f(x_2)$ where will the test points be on the next iteration?

x_1 = ______10.11145________ x_2 = _____8.94427_____________

On the marking scheme as used by the TA's and as originally posted the wrong wall was moved. Doing this gives x1 = 7.05572 and x2 = 5.88854. This answer (right idea, wrong wall) is worth one mark. Otherwise 1 mark for each value

Give full marks for x1 and x2 consistently reversed (different naming convention).

If a total of just five functions evaluations are permitted, what will E_{MAX} be at the end of the search?

E_{MAX} = ________0.583592____________________

Five function evaluations permit a total of four wall movements.
From the formula with $\Delta x^0 = 8$ and n = 4, $E_{MAX} = 0.583592$
1 mark for the correct answer.
Award 0.5 marks if student has the idea but has made a small slip (e.g. Δx^0 wrong or n = 5).