

MAT 1339 C
Calculus and Vectors
Test # 1
October 10, 2012

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Duration: 80 minutes

Name: _____

Student Number: _____

Instructions:

- Print your name and student number on this page.
- Verify that your copy of the exam has all 5 pages.
- There are 8 questions worth a total of 15 marks.
- You must answer all questions.
- Write your answers in the spaces below the questions. You may use the backs of the pages if necessary.
- **No Notes or Books.**
- **Basic scientific calculators only - graphing and/or programmable calculators are NOT permitted.**

GOOD LUCK

Multiple Choice Questions: (5 points) Circle the letter of the correct answer.

1. The average rate of change of $f(x) = x^2 + 3x - 5$ on $0 \leq x \leq 1$ is?

- (a) 4
- (b) 5
- (c) 6
- (d) 7

$$\frac{\Delta f}{\Delta x} = \frac{f(1) - f(0)}{1 - 0} = \frac{(1 + 3 - 5) - (-5)}{1} = \frac{4}{1} = 4$$

2. The limit $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x + 3}$ is?

- (a) does not exist
- (b) 2
- (c) 1
- (d) 0

Direct Substitution

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x + 3} = \frac{(3)^2 - 9}{3 + 3} = \frac{0}{6} = 0$$

3. If the demand (or price) function of a product is $p(x) = 32 - 0.02x$, what is the marginal revenue for the sale of 100 units?

- (a) \$36
- (b) \$28
- (c) \$32
- (d) \$30

$$R(x) = x p(x) = 32x - 0.02x^2$$

$$R'(x) = 32 - 0.04x$$

$$R'(100) = 32 - 0.04(100) = 32 - 4 = 28$$

4. Which of the following is NOT a valid differentiation rule?

- (a) $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$
- (b) $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
- (c) $(kf(x))' = kf'(x)$
- (d) $(f \circ g(x))' = f'(x)g'(x)$

5. If $f''(a) = 0$, the function $f(x)$ must have an inflection point at $x = a$.

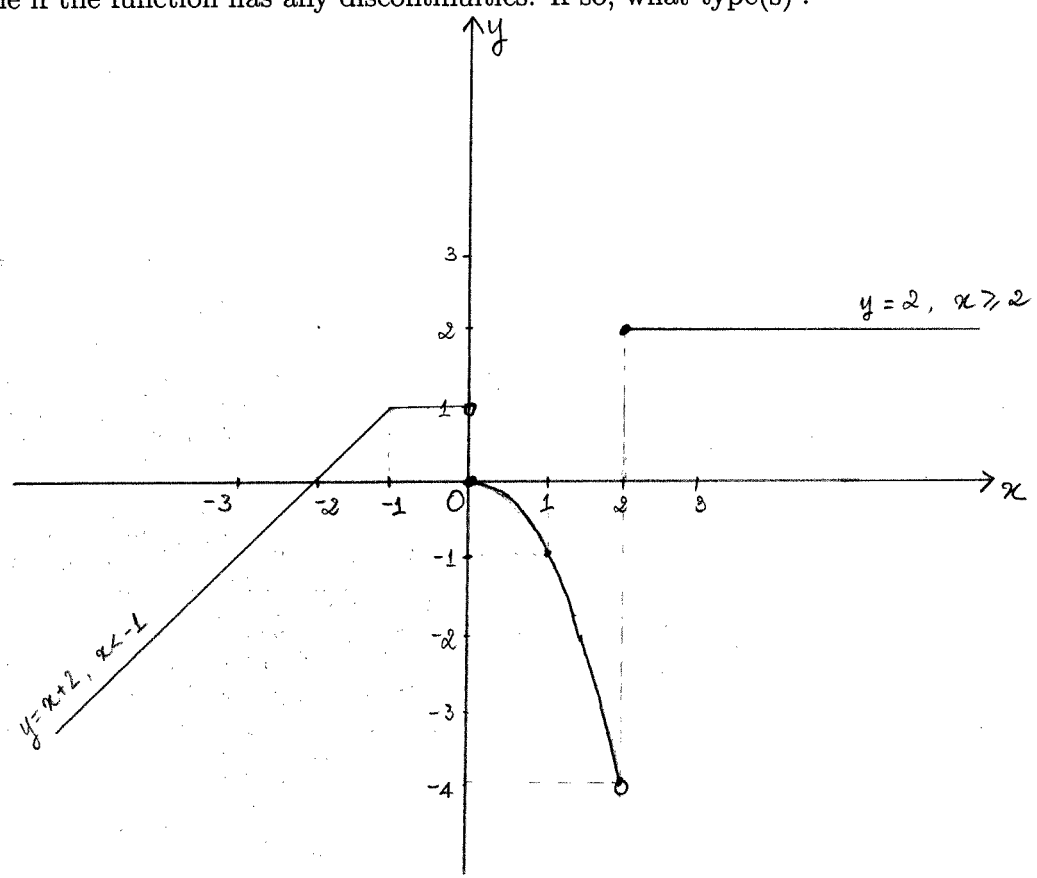
- (a) True
- (b) False

Counter-example : $y = f(x) = x^4$
 $y' = 4x^3$
 $y'' = f''(x) = 12x^2$
 $f''(0) = 0$ but $x = 0$ is not an inflection point

Problems: (10 points) Write your answers in the spaces provided.

1. (3 points) Consider the function $f(x) = \begin{cases} x + 2 & x < -1 \\ 1 & -1 \leq x < 0 \\ -x^2 & 0 \leq x < 2 \\ 2 & x \geq 2 \end{cases}$.

Determine if the function has any discontinuities. If so, what type(s)?



$f(x)$ has two jump discontinuities at $x = 0$ and $x = 2$

2. (3 points) Find the derivatives of the following functions. Simplify where possible.

(a) $f(x) = 3x^5 - 5x^2 + 7\sqrt{x} - \frac{3}{x^4}$

$$\begin{aligned}
f'(x) &= 15x^4 - 10x + 7\left(\frac{1}{2\sqrt{x}}\right) - (3x^{-4})' \\
&= 15x^4 - 10x + \frac{7}{2\sqrt{x}} + 12x^{-5} \\
&= 15x^4 - 10x + \frac{7}{2\sqrt{x}} + \frac{12}{x^5}
\end{aligned}$$

(b) $g(t) = (3t + 1)^4(4t^2 + 1)^5$

$$\begin{aligned}
g'(t) &= [(3t+1)^4]'(4t^2+1)^5 + (3t+1)^4[(4t^2+1)^5]' \\
&= 4(3t+1)^3(3)(4t^2+1)^5 + (3t+1)^4(5)(4t^2+1)^4(8t) \\
&= 12(3t+1)^3(4t^2+1)^5 + 40t(3t+1)^4(4t^2+1)^4 \\
&= 4(3t+1)^3(4t^2+1)^4 [3(4t^2+1) + 10t(3t+1)] \\
&= 4(3t+1)^3(4t^2+1)^4 (42t^2 + 10t + 3)
\end{aligned}$$

3. (4 points) Let $f(x) = x^3 + 3x^2 - 9x + 3$.

(a) Find the critical points of $f(x)$.

$$f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3)$$

$$= 3(x-1)(x+3)$$

$$f'(x) = 0 \Rightarrow 3(x-1)(x+3) = 0$$

$$x = 1 \quad | \leftarrow \text{critical points}$$

$$x = -3$$

(b) Find the intervals where $f(x)$ is increasing, decreasing.

Table:

x	$-\infty$		-3		1		∞
f'		$+$	0	$-$	0	$+$	
f		↗ L. Max.		↘ L. Min.			

Increasing: $(-\infty, -3) \cup (1, \infty)$

Decreasing: $(-3, 1)$

(c) Find the local maximum points and local minimum points of $f(x)$.

Local Max. at $x = -3$, $(-3, 30)$

Local Min at $x = 1$ $(1, -2)$

(d) Find the intervals where $f(x)$ is concave up, concave down.

$$f''(x) = 6x + 6 \quad , \quad f''(x) = 0 \Leftrightarrow x = -1$$

x	$-\infty$		-1		∞
f''		$-$	0	$+$	
f		down		up	

Concave up: $(-1, \infty)$, Concave down: $(-\infty, -1)$

(e) Find the inflection points of $f(x)$.

Inflection point at $x = -1$