MAT 1339 C Fall 2012 – Assignment #2
Due in class on Wednesday, October 17th

Name (print): SOLUTIONS

Student Number:

Instructions: Please print this assignment, enter your name and student number above, and answer all questions in the spaces provided. Your work must be shown to receive credit and all steps should be presented neatly, logically and clearly.

Question 1 The cost, in dollars, of producing x widgets is $C(x) = 0.1x^2 + 30x + 100$ and the demand or price function is $p(x) = 125 - 0.2x$.

(a) Find the revenue and profit functions.

$$R(x) = x\,p(x) = x(125 - 0.2x) = -0.2x^2 + 125x$$

$$P(x) = R(x) - C(x) = -0.2x^2 + 125x - (0.1x^2 + 30x + 100)$$

$$= -0.3x^2 + 95x - 100$$

(b) Determine the marginal cost at 100 widgets.

$$C'(x) = 0.2x + 30$$

$$C'(100) = 0.2(100) + 30 = 50$$

(c) Determine the actual cost of producing the 101st widget.

$$\Delta C = C(101) - C(100)$$

$$= 0.1(101)^2 + 30(101) + 100 - \left(0.1(100)^2 + 30(100) + 100\right)$$

$$= 50.10$$

(d) What are the marginal revenue and marginal profit for the sale of 100 widgets?

$$R'(x) = -0.4x + 125$$

$$R'(100) = -0.4(100) + 125 = 85$$

$$P'(x) = -0.6x + 95$$

$$P'(100) = -0.6(100) + 95 = 35$$

Question 2 For each of the following functions:
(i) find the intervals of increase and decrease and identify any local extrema and
(ii) find the intervals of concavity and any inflection points.

(a) $g(t) = 2t^3 + 15t^2 - 84t + 13$

$g'(t) = 6t^2 + 30t - 84 = 6(t^2+5t-14)$

$= 6(t+7)(t-2)$

$g'(t) = 0 \Leftrightarrow t = -7$ or $t = 2$.

Critical points : $t = -7$ and $t = 2$

t	$-\infty$		-7		2		∞
g'		$+$	0	$-$	0	$+$	
g		$\nearrow$		$\searrow$		$\nearrow$	

Increasing: $(-\infty,-7) \cup (2,\infty)$
Decreasing : $(-7,2)$
Local Max. at $t=-7$
Local Min. at $t=2$

$g''(t) = 12t+30 \Rightarrow g''(t)=0 \Leftrightarrow t=-\frac{5}{2} \Leftarrow$ inflection pts.

t	$-\infty$		$-5/2$		∞
g''		$-$	0	$+$	
g		down		up	

Concave up : $(-\frac{5}{2},\infty)$
Concave down : $(-\infty,-\frac{5}{2})$

(b) $y = \dfrac{5}{x+3}$

$y' = \dfrac{-5}{(x+3)^2} < 0$ for all $x \neq -3$

$\Rightarrow$ function is always decreasing and there is no critical point.

$y'' = \dfrac{10}{(x+3)^3} \neq 0 \quad \forall x \neq -3$

$y'' > 0 \Leftrightarrow (x+3)^3 > 0 \Leftrightarrow (x+3) > 0 \Leftrightarrow x > -3$

Concave up $(-3,\infty)$

$y'' < 0 \Leftrightarrow x < -3$

Concave down $(-\infty,-3)$

Question 3 Find the absolute max and min of the given function on the given interval.

(a) $f(x) = \dfrac{3}{x+2}$ on $[0, 5]$

$f'(x) = \dfrac{-3}{(x+2)^2} < 0$ for all $x \neq -2$

$\Rightarrow$ always decreasing

$f(0) = \dfrac{3}{2}$ $\leftarrow$ absolute max

$f(5) = \dfrac{3}{7}$ $\leftarrow$ absolute min

(b) $y = \dfrac{t^2}{2} + \dfrac{8}{t}$ on $[1, 4]$

$y' = t - \dfrac{8}{t^2} = \dfrac{t^3 - 8}{t}$, $y' = 0 \Leftrightarrow t^3 - 8 = 0 \Rightarrow t = 2$

Critical point $t = 2 \in [1, 4]$

$y(1) = \dfrac{1}{2} + \dfrac{8}{1} = \dfrac{17}{2}$

$y(2) = \dfrac{(2)^2}{2} + \dfrac{8}{2} = 6$ $\leftarrow$ absolute min

$y(4) = \dfrac{(4)^2}{2} + \dfrac{8}{4} = 10$ $\leftarrow$ absolute max

(c) $f(t) = t^4 - 8t^3 + 22t^2 - 24t$ on $[0, 4]$

$f'(t) = 4t^3 - 24t^2 + 44t - 24$

$= 4(t^3 - 6t^2 + 11t - 6) = 4(t-1)(t-2)(t-3)$

Critical points $t = 1$, $t = 2$, $t = 3$ in $[0, 4]$

$f(0) = 0$

$f(1) = -9$

$f(2) = -8$

$f(3) = -9$

$f(4) = 0$

Absolute max at $t = 0$ and $t = 4$

Absolute min at $t = 1$ and $t = 3$

Question 4 Identify any vertical asymptotes of the following functions.

(a) $f(x) = \dfrac{4}{x+6}$

$D = \mathbb{R} \setminus \{-6\}$

$\lim_{x \to -6} f(x) = \lim_{x \to -6} \left(\dfrac{4}{x+6} \right) = \infty \Rightarrow$ VA is $x = -6$

(b) $f(x) = \dfrac{x}{x^2+3}$

$D = \mathbb{R}$ since $x^2 + 3 \neq 0$ for all x

$\Rightarrow$ no VA

(c) $y = \dfrac{2t}{t^2-4} = \dfrac{2t}{(t-2)(t+2)}$

$D = \mathbb{R} \setminus \{-2, 2\}$

$\lim_{t \to -2} f(t) = \infty$ $\Rightarrow$ VA are $t = -2$ and $t = 2$

$\lim_{t \to 2} f(t) = \infty$

(d) $g(r) = \dfrac{3r+1}{r^2-4r-12} = \dfrac{3r+1}{(r-6)(r+2)}$

$D = \mathbb{R} \setminus \{-2, 6\}$

$\lim_{r \to -2} g(r) = \infty$ $\Rightarrow$ VA are $r = -2$

$\lim_{r \to 6} g(r) = \infty$ and $r = 6$