

MAT 1339 C Fall 2012 – Assignment #1
Due in class on Wednesday, September 26th

Name (print): _____

SOLUTIONS

Student Number: _____

Instructions: Please print this assignment, enter your name and student number above, and answer all questions in the spaces provided. Your work must be shown to receive credit and all steps should be presented neatly, logically and clearly.

Question 1 A city's population is given in the table.

Year	Population
2000	753 612
2001	754 806
2002	756 124
2003	758 070
2004	759 980
2005	761 246
2006	763 573
2007	765 667
2008	770 100
2009	772 675
2010	775 214

What is the average rate of change of the population in

(a) 2000 to 2004 ?

$$\frac{\Delta P}{\Delta t} = \frac{759980 - 753612}{2004 - 2000} = 1592 \quad (\text{people/year})$$

(b) 2003 to 2009 ?

$$\frac{\Delta P}{\Delta t} = \frac{772675 - 758070}{2009 - 2003} \approx 2434 \quad (\text{people/year})$$

Question 2 Evaluate the limits.

$$\begin{aligned} \text{(a)} \quad \lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2} &= \lim_{x \rightarrow 2} \frac{(x^2 - 4)(x^2 + 4)}{x - 2} \\ &= \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)(x^2 + 4)}{x - 2} \end{aligned}$$

$$= \lim_{x \rightarrow 2} (x + 2)(x^2 + 4) = \boxed{32}$$

$$\text{(b)} \quad \lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{x} = \lim_{x \rightarrow 0} \frac{(\sqrt{4+x} - 2)(\sqrt{4+x} + 2)}{x(\sqrt{4+x} + 2)}$$

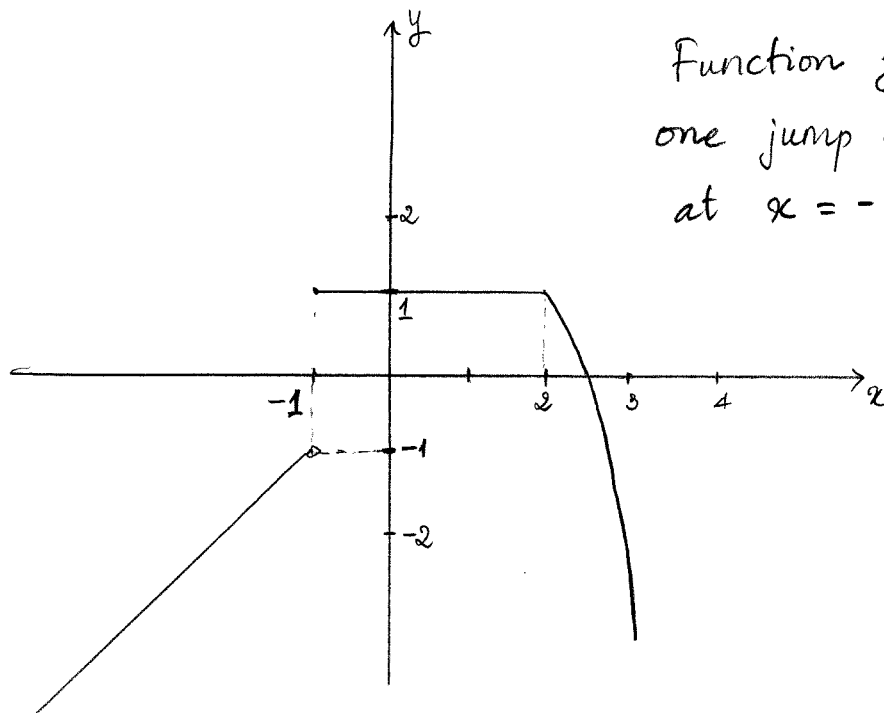
$$= \lim_{x \rightarrow 0} \frac{(4+x) - 4}{x(\sqrt{4+x} + 2)}$$

$$= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{4+x} + 2)}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{4+x} + 2} = \frac{1}{4}$$

Question 3 Graph the piecewise function $f(x) = \begin{cases} x & x < -1 \\ 1 & -1 \leq x \leq 2 \\ 5 - x^2 & x > 2 \end{cases}$.

Does f have any discontinuities? If so, what type and where?



Function $f(x)$ has
one jump discontinuity
at $x = -1$

Question 4 Use the definition of the derivative to find $f'(x)$ if $f(x) = \frac{2}{x^2}$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{2}{(x+h)^2} - \frac{2}{x^2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2x^2 - 2(x^2 + 2xh + h^2)}{hx^2(x+h)^2}$$

$$= \lim_{h \rightarrow 0} \frac{h(-2h - 4x)}{hx^2(x+h)^2}$$

$$= \lim_{h \rightarrow 0} \frac{-2h - 4x}{x^2(x+h)^2} = \frac{-4x}{x^4} = \frac{-4}{x^3}$$

Question 5 Find the derivatives of the following. Simplify where possible.

(a) $g(t) = 3t^2 - 7t + 2\sqrt{t}$

$$g'(t) = 6t - 7 + 2 \cdot \left(\frac{1}{2\sqrt{t}} \right)$$

$$= 6t - 7 + \frac{1}{\sqrt{t}}$$

(b) $f(x) = 2\pi x^4 - 3x^{2/3} + 2x^{-1/6}$

$$f'(x) = 2\pi(4x^3) - 3\left(\frac{2}{3}x^{-1/3}\right) + 2\left(-\frac{1}{6}x^{-7/6}\right)$$

$$= 8\pi x^3 - 2x^{-1/3} - \frac{1}{3}x^{-7/6}$$

(c) $y = (2x^2 + 3x)^3 (3x^4 + 6)^2$

$$= [(2x^2 + 3x)^3]' [(3x^4 + 6)^2] + [(2x^2 + 3x)^3] [(3x^4 + 6)^2]'$$

$$= 3(2x^2 + 3x)^2(4x + 3)(3x^4 + 6)^2 + (2x^2 + 3x)^3(2)(3x^4 + 6)(12x^3)$$

$$= 3(2x^2 + 3x)^2(3x^4 + 6)[(4x + 3)(3x^4 + 6) + 8x^3(2x^2 + 3x)]$$

(d) $f(x) = \sqrt{7x^4 + 6x^2 + 1}$ let $u = 7x^4 + 6x^2 + 1 \Rightarrow u' = 28x^3 + 12x$
 then $y = \sqrt{u} = u^{1/2} \Rightarrow y' = f'(x) = \frac{1}{2\sqrt{u}} u'$

$$f'(x) = \frac{1}{2\sqrt{7x^4 + 6x^2 + 1}} \cdot (28x^3 + 12x) = \frac{14x^3 + 6x}{\sqrt{7x^4 + 6x^2 + 1}}$$

Question 6 An object moving in a straight line has position function $s(t) = 2t^3 + 4t^2 - 6t + 2$ (in m for t in s). Determine the acceleration of the object at time $t = 2$ s.

Velocity $v(t) = s'(t) = 6t^2 + 8t - 6$

Acceleration $a(t) = v'(t) = 12t + 8$

so $a(2) = 12(2) + 8 = 32 \text{ (m/s}^2\text{)}$

Question 7 What is the equation of the tangent line to the graph of $y = f(x) = \frac{x+2}{x^2+4}$ at $x=0$? Equation of tangent line $y = f'(0)(x-0) + f(0)$

• $y(0) = f(0) = \frac{1}{2}$

• $y' = f'(x) = \frac{(x+2)'(x^2+4) - (x+2)(x^2+4)'}{(x^2+4)^2} = \frac{(x^2+4) - 2x(x+2)}{(x^2+4)^2}$

$y'(0) = f'(0) = \frac{4}{16} = \frac{1}{4}$

\Rightarrow Equation: $y = \frac{1}{4}(x-0) + \frac{1}{2}$ or $y = \frac{1}{4}x + \frac{1}{2}$