1. Find an equation of the plane which is perpendicular to the (x, y)-plane and which goes through the points $(1, 4, 1)$ and $(2, 0, 1)$.

A. x+4y+z=3

B. 4x+y-8z=0

C. x+4y=2

D. 4x+y=8

E. z=1

F. 2x-y+2z=0

Perp. to the (x,y) - plane means: z does not occur in the equation.

$(1,4,1)$ is not in the plane $x+4y=2$.

That leaves D.

(You can check both points are in it.)

2. Consider the two planes U given by $2x - y + z = 1$ and V given by $x + 3y - 4z = 4$. Which of the following is true?

A. U and V are parallel

B. U and V intersect in the line $(1, 1, 0) + t(1, 3, -4)$

C. U and V intersect in the plane $x - 2y + 5z = -1$

D. U and V intersect in the line $(1, 1, 0) + t(1, 9, 7)$

E. U and V intersect in the line $(1, 1, 0) + t(1, -2, -1)$

F. U and V intersect in the line $(2, -1, 1) + t(1, 3, -4)$

To find the direction vector, compute

$(2,-1,1) \times (1,3,-4) = (1,9,7)$.

Thus D is correct (you can check that $(1,1,0)$ is in both planes).

3. Find the point of intersection of the plane $x-2y-3z=-4$ and the line $(6,1,2)+t(-4,-3,1)$.

A. $(1,1,1)$
B. $(3,2,1)$
C. $(1,2,1)$
D. $(-2,-5,4)$
E. $(6,11,-11)$
F. None of the above.

Substitute $x=6-4t$, $y=1-3t$, $z=2+t$ into $x-2y-3z=-4$ to get

$$(6-4t)-2(1-3t)-3(2+t) = -4 \iff$$
$$-2-t=-4 \iff$$
$$t=2$$

The point then is $(6,1,2)+2(-4,-3,1)=(-2,-5,4)$.

4. Consider the vectors $u=(1,4,0)$ and $v=(2,2,-3)$. Which statement is true about the angle θ between u and v?

A. $\theta=\frac{10\pi}{17}$
B. $\cos\theta=\frac{10}{\sqrt{17}}$
C. $\cos\theta=\frac{1}{17}$
D. $\cos\theta=\frac{10}{17}$
E. $\cos\theta=\sqrt{10/17}$
F. $\theta=10$

$$\cos\theta = \frac{u\cdot v}{\|u\|\cdot\|v\|}$$
$$= \frac{1\cdot 2+4\cdot 2+0\cdot(-3)}{\sqrt{1^2+4^2+0^2}\cdot\sqrt{2^2+2^2+(-3)^2}}$$
$$= \frac{10}{\sqrt{17}\cdot\sqrt{17}}$$
$$= \frac{10}{17}$$

5. Find the volume of the parallellepipid determined by the vectors $\underset{u}{(1,1,2)}$, $\underset{v}{(3,2,-1)}$, $\underset{w}{(3,3,-1)}$.

A. 0

B. 4

C. 7

D. 12

E. 21

F. None of the above.

Compute $|u \cdot (v \times w)|$.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -1 \\ 3 & 3 & -1 \end{vmatrix} = 1 \cdot \hat{i} + 0\hat{j} + 3k = (1,0,3)$$

$(1,1,2) \cdot (1,0,3) = 7$

6. Which of the following statements is *not* correct?

A. $(av) \times v = 0$ for all vectors $v \in \mathbb{R}^3$ and all scalars $a \in \mathbb{R}$

B. $v \times u = -u \times v$ for all vectors $u, v \in \mathbb{R}^3$

C. $\frac{u \cdot v}{||u||\,||v||} \leq 99$ for all vectors $u, v \in \mathbb{R}^3$

D. $Proj_v(u) \cdot v = 0$ for all vectors $u, v \in \mathbb{R}^3$

E. $u - Proj_v(u)$ is orthogonal to v for all vectors $u, v \in \mathbb{R}^3$

F. There exist vectors $u, v, w \in \mathbb{R}^3$ such that $u \times (v \times w) \neq (u \times v) \times w$

D is false: $Proj_v(u)$ is parallel to v; thus if u, v are not orthogonal, it is nonzero

(Ex $u = (0,0,1)$ $v = (0,1,1)$; then

$$Proj_v(u) = \frac{(0,0,1) \cdot (0,1,1)}{(0,1,1) \cdot (0,1,1)} \cdot v = \frac{1}{2} v = (0, \tfrac{1}{2}, \tfrac{1}{2})$$

So $proj_v(u) \cdot v = (0,1,1) \cdot (0, \frac{1}{2}, \frac{1}{2}) = 1 \neq 0$.)

7. Let M be the plane through $(1,4,1),(2,2,3)$ and $(0,0,0)$. Find the equation of the line through $(1,1,1)$ perpendicular to M.

A. $(1,-2,2)+t(10,-1,-6)$

B. $(1,1,1)+t(1,-2,2)$

C. $(1,-2,2)+t(1,1,1)$

D. $(10,-1,-6)+t(1,-2,2)$

E. $(11,0,-5)+t(10,-1,-6)$

F. None of the above

The normal of M can be found as $(1,4,1)\times(2,2,3) = (10,-1,-6)$.

Thus either A or E is correct.

Now $(1,1,1)+(10,-1,-6) = (11,0,-5)$

So $(11,0,-5)$ is on the line.

8. An equation for the plane with parametric representation $(1,1,1)+t(1,2,-2)+s(0,3,1)$ is

A. $2x+6y=8$

B. $8x-y+3z=10$

C. $x+y+z=3$

D. $x+y+z=1$

E. $4x-3y+2z=3$

F. $3y+z=1$

The normal is found by

$(1,2,-2)\times(0,3,1) = (8,-1,3)$

Check that $(1,1,1)$ is in it:

$8\cdot 1 - 1\cdot 1 + 3\cdot 1 = 10$,

So B is correct

9. Find the area of the triangle with vertices $(1, -3, 1)$, $(2, 0, 3)$, $(4, 1, 4)$.

A B C

A. 1.5

B. 3

C. 4.5

D. $\sqrt{11}$

E. $\sqrt{35}$

F. None of the above

$$\overrightarrow{AB} = (1, 3, 2)$$
$$\overrightarrow{AC} = (3, 4, 3)$$
$$\overrightarrow{AB} \times \overrightarrow{AC} = (1, 3, -5)$$
$$\|\overrightarrow{AB} \times \overrightarrow{AC}\| = \sqrt{1^2 + 3^2 + (-5)^2} = \sqrt{35}$$

area of parallellogram.

The area of the $\triangle$ is $\frac{1}{2}\sqrt{35}$

10. Find the orthogonal projection of $(2, 5, 1)$ on $(2, -1, -1)$.

u v

A. $(-\frac{2}{5}, \frac{1}{5}, \frac{1}{5})$

B. $(-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}})$

C. $(-12, 6, 6)$

D. $(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$

E. $(-\frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

F. None of the above.

$$\text{Proj}_v(u) = \frac{u \cdot v}{v \cdot v} \cdot v$$
$$= \frac{(2,5,1) \cdot (2,-1,-1)}{(2,-1,-1) \cdot (2,-1,-1)} \cdot (2,-1,-1)$$
$$= \frac{2 \cdot 2 + 5 \cdot (-1) + 1 \cdot (-1)}{2 \cdot 2 + (-1) \cdot (-1) + (-1) \cdot (-1)} \cdot (2,-1,-1)$$
$$= \frac{-2}{6} (2, -1, -1)$$
$$= (-2/3, 1/3, 1/3)$$