University of Waterloo
Department of Electrical and Computer Engineering
SPRING TERM 2009

ECE261: ENERGY SYSTEMS
Mid-Term Examination

Date: Friday, 19th June 2009
Maximum Marks- 45
Time: 3:00 – 4:20 PM
Contribution to Overall Course Grade = 25%
Examiner: Professor Kankar Bhattacharya

Sample
KB

Full Name: ______________________

ID Number: ______________________

Note:

1. Write your ID number on each page of this booklet.
2. Answer ALL questions. This is a closed book exam. Only scientific (non-programmable) electronic calculators may be used.
3. A list of formulas is provided at the end of this exam paper. You may detach the page for convenience (it will not be marked- do not write anything on it).
4. Show all worked out steps clearly in the solution. No marks will be awarded for numerical results unless correct steps are provided. Further note that, numerical accuracy is also equally important.
5. **Provide correct SI units with all your answers.**
6. **Write Clean! It will help you!**

Question	1 (12)	2 (10)	3 (18)	4 (5)	TOTAL (45)
Marks Obtained					

GOOD LUCK!

Q.1: Three Phase Systems **(12 marks)**

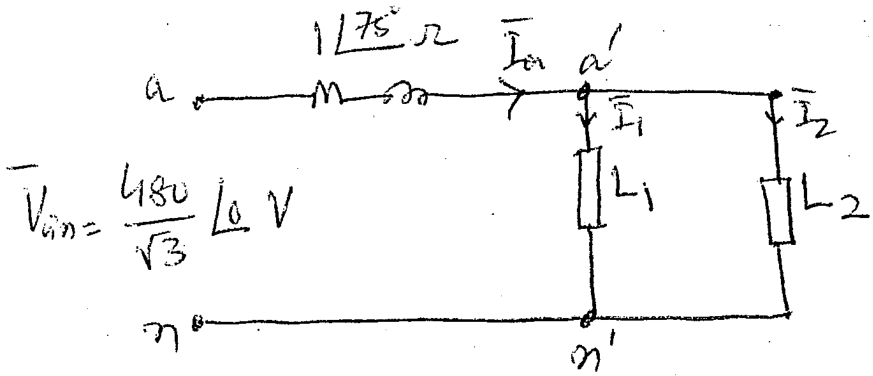
A balanced, 3-phase, 480 V, distribution system supplies power to two loads connected in parallel, via a transmission line of impedance $1\angle 75°$ ohms per phase. The two loads are as follows: a) Load-1 is a balanced Δ-connected load with a per phase impedance of $9\angle 30°$ ohms and b) Load-2 is a balanced Y-connected load with a per phase impedance of $5\angle -36.87°$ ohms.

a. Find the current in each phase of Load-1 and Load-2 (6)

b. Find the current in the lines feeding the three-phase loads (3)

c. Find the total active and reactive power supplied by the source (3)

Single phase equivalent circuit is as follows:



$$L_{1Y} = \frac{1}{3} L_{1\Delta} = \frac{1}{3}(9\angle 30°)\ \Omega = 3\angle 30°\ \Omega$$

$$L_{2Y} = 5\angle -36.87°\ \Omega$$

$$\bar{I}_a = \frac{\bar{V}_{an}}{\bar{Z}_L + Z_{L_1} \| Z_{L_2}}$$

$$= \frac{480/\sqrt{3}\angle 0°}{1\angle 75° + \dfrac{(3\angle 30°)(5\angle -36.87°)}{3\angle 30° + 5\angle -36.87°}} = \frac{277.13\angle 0°\,[3\angle 30° + 5\angle -36.87°]}{3\angle 105° + 5\angle 38.13° + 15\angle -6.87}$$

$$= \frac{1828.52 - j415.697}{18.05 + j4.19} = \frac{1875.18\angle -12.81}{18.53\angle 13.07} = 101.2\angle -25.9° \text{ A}$$

Thus, $\bar{I}_1 = \bar{I}_a \dfrac{L_2}{L_1 + L_2}$, $\bar{I}_2 = \bar{I}_a \dfrac{L_1}{L_1+L_2}$

$$\bar{I}_1 = 101.2 \angle -25.9^\circ \left[\frac{5 \angle -36.87^\circ}{3 \angle 30^\circ + 5 \angle -36.87^\circ} \right] = 74.764 \angle -49.97^\circ \text{ A}$$

$$\bar{I}_2 = 101.2 \angle -25.9 \left[\frac{3 \angle 30^\circ}{3 \angle 30^\circ + 5 \angle -36.387^\circ} \right] = 44.858 \angle 16.9^\circ \text{ A}$$

$$\bar{V}_{a'n'} = \bar{V}_{an} - \bar{I}_a \bar{Z}_L = \frac{480}{\sqrt{3}} \angle 0^\circ - (101.2 \angle -25.9)(1 \angle 75^\circ)$$

$$= 277.13 \angle 0^\circ - 101.2 \angle 49.1^\circ$$

$$= 224.31 \angle -19.94^\circ$$

Therefore, $\bar{V}_{a'b'} = \sqrt{3}\, \bar{V}_{a'n'} \angle 30^\circ = \sqrt{3} \left[224.31 \angle -19.94^\circ \right] \angle 30^\circ \text{ V}$

$$= 388.52 \angle 10.06^\circ \text{ V}$$

Thus, $\bar{I}_{a'b'} = \dfrac{\bar{V}_{a'b'}}{\bar{Z}_\Delta} = \dfrac{388.52 \angle 10.06^\circ}{9 \angle 30^\circ} = 43.17 \angle -19.94^\circ \text{ A}$

$$\bar{I}_{b'c'} = 43.17 \angle -139.94^\circ \text{ A}, \quad \bar{I}_{c'a'} = 43.17 \angle 100.06^\circ \text{ A}$$

$\bar{I}_{a'b'}$, $\bar{I}_{b'c'}$ and $\bar{I}_{c'a'}$ are the currents in each phase of Load - 1.

We also have, $\bar{I}_2 = 44.858 \angle 16.9^\circ$ A. Since Load-2 is Y-connected, $\bar{I}_2 = \bar{I}_{a'n}$.

Hence $\bar{I}_{a'n'} = 44.858 \angle 16.9^\circ$ A

$\bar{I}_{b'n'} = 44.858 \angle -103.1^\circ$ A, $\bar{I}_{c'n'} = 44.858 \angle 136.9^\circ$ A

$\bar{I}_{a'n'}$, $\bar{I}_{b'n}$ and $\bar{I}_{c'n'}$ are the currents in each phase of Load-1

b) Line currents:

$$\bar{I}_a = 101.2 \angle -25.9^\circ \text{ A}$$

$$\bar{I}_b = 101.2 \angle -145.9^\circ \text{ A}, \quad \bar{I}_c = 101.2 \angle 94.1^\circ \text{ A}$$

c) Complex Power Supplied by Source:

$$S = 3 \bar{V}_{an} \bar{I}_a^* = 3 \left[\frac{480}{\sqrt{3}} \angle 0^\circ \right] \left[101.2 \angle +25.9^\circ \right]$$

$$= 84136.1 \angle 25.9^\circ \text{ VA}$$

$$= 75685.3 \text{ W} + 36750.8 \text{ VAR}$$

$$\left. \begin{array}{l} P = 75.685 \text{ kW} \\ Q = 36.75 \text{ KVAR} \end{array} \right\} \text{Supplied from Source.}$$

Q.2: Magnetic Circuits **(10 marks)**

A magnetic core has a circular cross-sectional area of 2 cm^2. It has a mean path length of 10 cm and an air-gap length of 0.125 cm. A 350-turn coil is wound around the magnetic core, and a current of 5 A is supplied to the coil. The relative permeability of the core is 5000. There is a 5% fringing effect of the magnetic flux in the air-gap. Calculate:

a. Reluctance of the magnetic circuit (5)

b. Magnetic flux density in the air-gap (5)

$A_c = 2\ cm^2 = 2\times10^{-4}\ m^2$

$l_c = 10\ cm = 0.1\ m$

$l_g = 0.125\ cm = 0.00125 m$

$N = 350t$

$I = 5A$

$\mu_r = 5000$

$fringing = 5\%$

a) $R_c = \dfrac{l_c}{\mu_r . A_c} = \dfrac{0.1\ m}{5000 \times 4\pi\times10^{-7}\ \frac{Wb}{At\text{-}m} \times 2\times10^{-4}\ m^2}$

$= 79577\ At/Wb$

$R_g = \dfrac{l_g}{\mu_o A_g} = \dfrac{0.00125\ m}{4\pi\times10^{-7}\ \frac{Wb}{At\text{-}m} \times (2\times1.05\times10^{-4}\ m^2)}$

$= 473675h.26\ At/Wb$

$R = R_c + R_g = 4,816,331.26\ At/Wb$

b) $N \cdot I = \mathcal{R}\phi$

$$\phi = \frac{350 \times 5 \text{ At}}{4816331 \cdot 26 \text{ At/Wb}} = 363 \cdot 347 \times 10^{-6} \text{ Wb}$$

$$B_{air\text{-}gap} = \frac{\phi}{A_g} = \frac{363 \cdot 347 \times 10^{-6} \text{ Wb}}{1 \cdot 05 \times 2 \times 10^{-4} \text{ m}^2}$$

$$= 1 \cdot 73 \text{ Wb/m}^2$$

$$= 1 \cdot 73 \text{ Tesla}$$

Q.3 Transformers

(18 marks)

A 50 kVA, 2400 / 240V, single phase transformer is tested and the following test data were obtained:

Meter Readings	Open circuit test	Short circuit test
Voltmeter	240 V	55 V
Ammeter	5.0 A	20.8 A
Wattmeter	450 W	600 W

a. Find the equivalent circuit parameters referred to the high voltage side. **(6)**

b. Calculate the voltage regulation when the transformer is connected to a load that takes 156 A at 220 V and 0.8 power factor leading. **(6)**

c. Find the copper loss, core loss and hence the transformer efficiency for the same loading condition of (b). **(6)**

From OCT :

$$|Y_E| = \frac{I_{oc}}{V_{oc}} = \frac{5}{240} = 0.020833\ \mho$$

$$\theta = \cos^{-1}\left(\frac{P_{oc}}{V_{oc}\cdot I_{oc}}\right) = \cos^{-1}\left(\frac{450}{240\times 5}\right) = 67.976^\circ$$

$$\bar{Y}_E = 0.020833\angle{-67.976^\circ}\ \mho$$

$$= 0.0078123 - j\,0.019313 = G_c - j\,B_m$$

$$\Rightarrow R_c = \frac{1}{G_c} = \frac{1}{0.0078123} = 128.0033\ \Omega$$

$$X_m = \frac{1}{B_m} = \frac{1}{0.019313} = 51.7786\ \Omega$$

The above parameters are referred to the L.V. Side. Thus,

$$R_{c\,HV} = a^2 R_c = (10)^2(128.0033) = 12800\ \Omega$$

$$X_{m\,HV} = a^2 X_m = (10)^2(51.7786) = 5178\ \Omega$$

from SCT

$$|Z_{e_1}| = \frac{V_{sc}}{I_{sc}} = \frac{55}{20.8} = 2.644\ \Omega$$

$$\theta = \cos^{-1}\left(\frac{P_{sc}}{V_{sc}\cdot I_{sc}}\right) = \cos^{-1}\left(\frac{600}{55 \times 20.8}\right) = 58.367^\circ$$

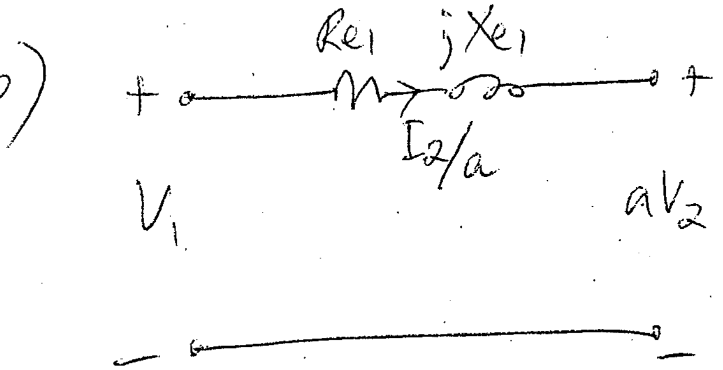
$$\bar{Z}_{e_1} = 2.644\angle 58.367^\circ = 1.3867 + j2.2517\ \Omega$$

$$\bar{Z}_{e_1} = R_{e_1} + jX_{e_1} \Rightarrow R_{e_1} = 1.3867\ \Omega$$

$$X_{e_1} = 2.2517\ \Omega$$

The above parameters are already referred to HV side.

b)



$$\bar{I}_2 = 156\angle +\cos^{-1}(0.8)\ A$$

$$= 156\angle 36.87^\circ\ A$$

This is the load current in the LV side.

$$\bar{V}_1 = a\bar{V}_2 + \frac{\bar{I}_2}{a}\left(R_{e_1} + jX_{e_1}\right)$$

$$= 10\left(220\angle 0^\circ\right) + \frac{156}{10}\angle 36.87^\circ\left(1.3867 + j2.2517\right)$$

$$= 2200\angle 0^\circ + \left(15.6\angle 36.87^\circ\right)\left(2.644\angle 58.373^\circ\right)$$

$$= 2200\angle 0^\circ + 41.2464\angle 95.243^\circ = 2196.23 + j41.074$$

$$= 2196.614\angle 1.071^\circ\ V$$

Voltage Regulation : $\dfrac{|\bar{V}_1| - |a\bar{V}_2|}{|a\bar{V}_2|} \times 100$

$$= \frac{2196 \cdot 614 - 2200}{2200} \times 100$$

$$= -0 \cdot 154 \%$$

c) Output Power, $P_{out} = V_2 I_2 \cos\theta$

$$= 220 \times 156 \times 0 \cdot 8 = 27 \cdot 456 \text{ kW}$$

$$P_{Loss\,Core} = \frac{V_1^2}{Rc_{HV}} = \frac{(2196 \cdot 614)^2}{12800} = 376 \cdot 962 \text{ Watts}$$

$$P_{Loss\,copper} = (I_2/a)^2 Re_1 = (15 \cdot 6)^2 (1 \cdot 3867)$$

$$= 337 \cdot 467 \text{ Watts}$$

$$\eta = \frac{P_{output}}{P_{output} + P_{Loss\,Core} + P_{Loss\,Copper}} = \frac{27456 \times 100}{27456 + 376 \cdot 962 + 337 \cdot 46}$$

$$= 97 \cdot 464 \%$$

Question-4: Answer the following in brief **(1.25 x 4 = 5)**

1. What do you understand by the term "balanced" in a *balanced 3-phase system*?

- Loads in all the three phases are equal
- All the line impedances are equal
- Supply source emf in each phase are equal in magnitude, separated by 120°

2. Find the phasor representation of the sinusoid:

$$v(t) = 200\sin(wt + 40°)$$

$$\Rightarrow v(t) = 200 \sin(wt + 90° - 50°)$$
$$= 200 \cos(wt - 50°)$$

Hence $\bar{V} = \frac{200}{\sqrt{2}} \angle -50° = 141.42 \angle -50° \text{ V}$

3. Consider a positive phase sequence, 3-phase circuit. Given that the current in the a-b phase in a Δ-connected load is 15∠15° A. What is the current in line-a?

$$\bar{I}_{ab} = 15\angle 15° \text{ A}$$

$$\Rightarrow \bar{I}_a = (\sqrt{3}\angle -30°)\bar{I}_{ab} = (\sqrt{3} \times 15)\angle -30° + 15° \text{ A}$$
$$= 25.98\angle -15° \text{ A}$$

4. Given a load of 10 - j15 ohms. What is the power factor of this load? And state whether the reactive power component of the load is inductive or capacitive.

$$\text{power factor} = \cos\left(\tan^{-1}\left(\frac{15}{10}\right)\right) = 0.554 \text{ Leading}$$

[the negative sign with the complex component of load impedance denotes leading power factor]

The reactive power component of the load is capacitive.

Formula Sheet

3-Phase Circuits

- Y-Connection

  Relation of phase voltage and line voltage-

$$\begin{array}{lcl} \bar{V}_a = V_m\angle 0° & & \bar{V}_{ab} = \left(\sqrt{3}\angle 30°\right)\cdot \bar{V}_a \\ \bar{V}_b = V_m\angle -120° & \Rightarrow & \bar{V}_{bc} = \left(\sqrt{3}\angle 30°\right)\cdot \bar{V}_b \\ \bar{V}_c = V_m\angle 120° & & \bar{V}_{ca} = \left(\sqrt{3}\angle 30°\right)\cdot \bar{V}_c \end{array}$$

- Δ-Connection

  Relation of phase current and line current-

$$\begin{array}{lcl} \bar{I}_{AB} = I_{ph}\angle -\theta_Z & & \bar{I}_A = \left(\sqrt{3}\angle -30°\right)\cdot \bar{I}_{AB} \\ \bar{I}_{BC} = I_{ph}\angle\left(-\theta_Z - 120°\right) & \Rightarrow & \bar{I}_B = \left(\sqrt{3}\angle -30°\right)\cdot \bar{I}_{BC} \\ \bar{I}_{CA} = I_{ph}\angle\left(-\theta_Z - 240°\right) & & \bar{I}_C = \left(\sqrt{3}\angle -30°\right)\cdot \bar{I}_{CA} \end{array}$$

Magnetic Aspects of Machines

- $B = \Phi/A$, $\Im = N.I$, $H = \Im/l$, $\Im = \Re.\Phi$ $B = \mu H$
- $\Re = \dfrac{l}{\mu A} = \dfrac{l}{\mu_o \mu_r A}$ At/Wb, $\mu_o = 4\pi \times 10^{-7}$ Wb/At-m
- $\Re_{series} = \Re_1 + \Re_2 + ... + \Re_n$

Transformers:

1. Voltage Regulation:

$$\frac{\left|\bar{V}_{2,NL}\right| - \left|\bar{V}_{2,FL}\right|}{\left|\bar{V}_{2,FL}\right|} \times 100\% = \frac{\left|\bar{V}_1\right| - \left|a\bar{V}_2\right|}{\left|a\bar{V}_2\right|} \times 100\% = \frac{\left|\bar{V}_1/a\right| - \left|\bar{V}_2\right|}{\left|\bar{V}_2\right|} \times 100\%$$

2. O. C. Test-

$$\left|Y_E\right| = \frac{I_{oc}}{V_{oc}}; \quad \angle\theta = \cos^{-1}\left(\frac{P_{oc}}{V_{oc} I_{oc}}\right)$$

$$\bar{Y}_E = \left|Y_E\right|\angle -\theta = G_c - jB_m = \frac{1}{R_c} - j\frac{1}{X_m}$$

5. S. C. Test-

$$R_e = \frac{P_{sc}}{I_{sc}^2}; \quad Z_e = \frac{V_{sc}}{I_{sc}}; \quad X_e = \sqrt{Z_e^2 - R_e^2}$$

6. Efficiency

$$\eta = \frac{P_{Output}}{P_{Output} + \sum Loss}$$