MAT 2377 (Winter 2014)

Assignment 5
Due on April 4 in boxes in Mathematics department at 3pm.

Note : You must give complete details in your solutions. To receive points for the question, you must clearly justify your final answer.

1. Please answer the following 3 questions from the textbook.

   9.10, 9.46, 9.94

2. Five samples of the material were tested in a structure, and the average interior temperatures (in Celsius) reported were as follows :

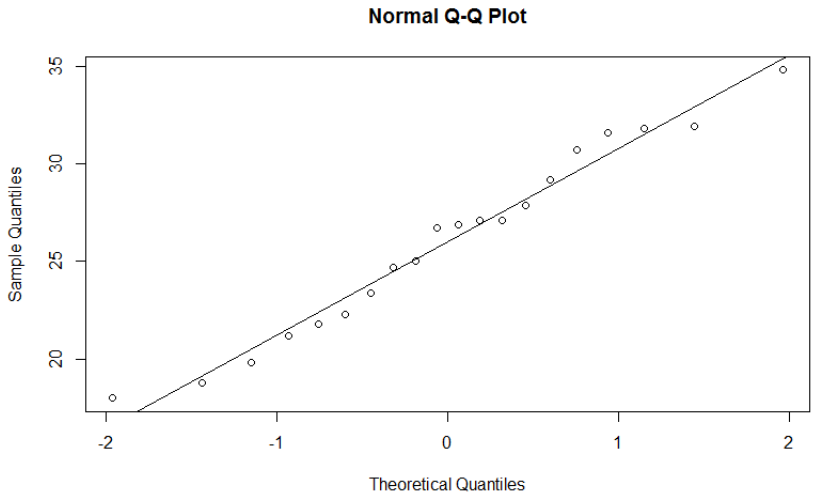
   $$23.01, 22.22, 22.04, 22.62, 22.59.$$

   Assume that the temperature is normally distributed.

   (a) Use a critical region to test the hypotheses $H_0 : \mu = 22.5$ against $H_1 : \mu \neq 22.5$, at a significance level of $\alpha = 5\%$.

   (b) Find the p-value corresponding to the test in part (a).

3. Cloud seeding has been studied for many decades as a weather modification procedure. The rainfall in acre-feet for 20 clouds that were selected at random and seeded with silver nitrate as been assigned to the variable x in R. With these data, we constructed the following quantile-quantile plot.



We computed a few statistics with R.

```
> summary(x)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  18.00   22.18   26.80   26.04   29.58   34.80
> sd(x)
[1] 4.784765
> length(x) ## sample size
[1] 20
```

(a) Based on the quantile-quantile plot, is is reasonable to assume that the rainfall is normally distributed ?

(b) Assuming that the rainfall is normally distributed, do we have significant evidence that the mean rainfall from seeded clouds is larger than 25 acre-feet ? Use a critical region with $\alpha = 0.01$.

(c) Compute the p-value that corresponds to the test in part (b).

4. **[R Problem] :** Perform the following steps in R to build a response variable y and a predictor x. The commands are found below.

(a) Generate a sample of 50 observations from a uniform distribution taken values between 0 and 10. Assign the values to the variable x.

(b) Generate normal random errors. We will generate 50 observations from the normal distribution with mean 0 and standard deviation 1.7. Assign the values to the variable e.

(c) Calculate the responses according to the following simple regression model $y = 1 + 3\,x + \varepsilon$, where $\varepsilon \sim N(0; (1.7)^2)$. Assign the responses to the variable y.

```
x=runif(50,0,10)        # Generate 50 values from a U(0,10) dist
e=rnorm(50,0,1.7)       # Generate 50 random errors
y=1+3*x+e               # Generate 50 responses
```

Answer the following questions :

(a) Construct the scatterplot of the points (x_i, y_i), for $i = 1, 2, \ldots, 50$ and with an overlay of the simple regression line. Describe the association between x and y.

```
plot(x,y)          # scatter plot
abline(lm(y~x))    # superimpose the fitted line
```

(b) Construct the scatterplot of the points (x_i, y_i), for $i = 1, 2, \ldots, 50$ and with an overlay of the simple regression line. Describe the association between x and y.

```
plot(x,y)          # scatter plot
abline(lm(y~x))    # superimpose the fitted line
```

(c) Perform a regression analysis of y as a function of x. Include the results of the regression analysis and identify the following : the residual sum of squares, the residual standard deviation, and the coefficient of determination.

```
summary(lm(y~x))  # perform a simple linear regression
```

(d) Write down the estimated regression line and use this line to predict a new response when $x = 2.75$.