1330hw7

MAT 1330, Fall 2011 Assignment 7

Due Wed December 7, at the beginning of class.

Late assignments will **not** be accepted; **nor** will unstapled assignments.

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Student Name ____________________ Student Number ____________________

By signing below. you declare that this work was your own and that you have not copied from any other individual or other source.

Signature ____________________

QUESTION 1.

Use the Newton's method to estimate the solution of the equation

$$2\cos(x) - x = 0.$$

by completing the following steps:

(a) Use the Intermediate Value Theorem to show that there exists a solution between 0 and $\frac{\pi}{2}$:

$f(x) = 2\cos(x) - x$

$f(0) = 2\cos(0) - 0 = 2 > 0$

$f(\frac{\pi}{2}) = 2\cos(\frac{\pi}{2}) - \frac{\pi}{2} = -\frac{\pi}{2} < 0$, then there is atleast one number c in $[0, \frac{\pi}{2}]$ such that $f(c) = 0$.

(b) Perform four iterations of the Newton's method with the initial value $x_0 = \frac{\pi}{4}$ (use 8 decimal places).

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$, where $f'(x) = -2\sin(x) - 1$

$x_{n+1} = x_n - \dfrac{2\cos(x_n) - x_n}{(-2\sin(x_n) - 1)} = x_n + \dfrac{2\cos(x_n) - x_n}{2\sin(x_n) + 1}$

$x_0 = \frac{\pi}{4} \approx 0.78539816$

n	x_n	x_{n+1}
0	0.78539816	1.04586203
1	1.04586203	1.02991392
2	1.02991392	1.02986653
3	1.02986653	1.02986653

QUESTION 2. Suppose that the length of a snake at age t is given by the function $L(t)$, which satisfies the following equation:

$$\frac{dL}{dt} = e^{-0.1t}, \quad t \geq 0.$$

(a) Find $L(t)$ if the limiting length L_∞ is given by

$$L_\infty = \lim_{t\to\infty} L(t) = 25 \text{ (inches)}.$$

$$L(t) = \int e^{-0.1t}dt = \frac{e^{-0.1t}}{-0.1} + C =$$

$$= -10e^{-0.1t} + C;$$

$$L(t) = -\frac{10}{e^{0.1t}} + C$$

$$L(\infty) = \lim_{t\to\infty} L(t) = \lim_{t\to\infty}\left(-\frac{10}{e^{0.1t}} + C\right) = C = 25$$

Thus, $L(t) = -10e^{-0.1t} + 25$

(b) How long was the snake at age $t = 0$?

$$L(0) = -10e^{-0.1\cdot 0} + 25 = 15 \text{ (inches)}$$

$$L(0) = -10 \cdot e^{-0.1\cdot 0} + 25 = -10 + 25 =$$
$$= 15 \text{(inches)}$$

Question 3.

Evaluate the following integrals.

(a) $\int x \ln x dx$ $\qquad \boxed{\int u dv = uv - \int v du}$

$$\boxed{\begin{array}{ll} \ln x = u & x dx = dv = v' dx \\ \frac{dx}{x} = du & x = v' \\ & v = \int x dx = \frac{x^2}{2} \end{array}}$$

$$\int x \ln x dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \cdot \frac{dx}{x} = \frac{x^2}{2} \ln x -$$
$$- \int \frac{x}{2} dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C.$$

(b) $\int \frac{dx}{x \ln x}$

$$\ln x = u$$
$$\frac{dx}{x} = du$$
$$\int \frac{dx}{x \ln x} = \int \frac{du}{u} = \ln|u| + C =$$
$$= \ln|\ln(x)| + C$$

(c) ~~$\int e^x \cos x \, dx$~~ $\quad \int e^{2x} \cos(3x) dx$

$$\boxed{\begin{array}{ll} \cos(3x) = u & e^{2x} dx = dv = v' dx \\ -3 \sin(3x) dx = du & \frac{e^{2x}}{2} = v \end{array}}$$

$$\cos(3x)\cdot\frac{e^{2x}}{2}-\int\frac{e^{2x}}{2}(-3\sin(3x))\,dx=\cos(3x)\frac{e^{2x}}{2}+$$

$$+\frac{3}{2}\int e^{2x}\sin(3x)\,dx=\boxed{\begin{matrix}\sin(3x)=u & e^{2x}dx=dv\\ 3\cos(3x)dx=du & \frac{e^{2x}}{2}=v\end{matrix}}=$$

$$=\cos(3x)\frac{e^{2x}}{2}+\frac{3}{2}\left[\sin(3x)\cdot\frac{e^{2x}}{2}-\int\frac{e^{2x}}{2}\cdot 3\cos(3x)\,dx\right]=$$

$$=\cos(3x)\frac{e^{2x}}{2}+\frac{3}{4}\sin(3x)e^{2x}-\frac{9}{4}\int e^{2x}\cos(3x)\,dx\Rightarrow$$

$$\frac{13}{4}=\left(1+\frac{9}{4}\right)\int e^{2x}\cos(3x)\,dx=\frac{e^{2x}}{2}\cos(3x)+\frac{3}{4}e^{2x}\sin(3x)\Rightarrow$$

(d) $\int x\sqrt{2x-1}\,dx$

$$\boxed{\int e^{2x}\cos(3x)\,dx=\frac{2}{13}e^{2x}\cos(3x)+\frac{3}{13}e^{2x}\sin(3x)}$$

Method 1: (Method 2, see, the last page)

$2x-1=u(x)\Rightarrow 2x=u+1\Rightarrow\boxed{x=\frac{u+1}{2}}$

$2dx=du\Rightarrow dx=\frac{du}{2}$

$$\int x\sqrt{2x-1}\,dx=\int\frac{u+1}{2}\cdot\sqrt{u}\,\frac{du}{2}=\frac{1}{4}\int\left(u^{\frac{3}{2}}+u^{\frac{1}{2}}\right)du=$$

$$=\frac{1}{4}\left[\frac{2u^{\frac{5}{2}}}{5}+\frac{2u^{\frac{3}{2}}}{3}\right]+C=\frac{1}{10}(2x-1)^{\frac{5}{2}}+\frac{1}{6}(2x-1)^{\frac{3}{2}}+C.$$

(e) $\int 3x^2\cos(x^3)\,dx$

$x^3=u(x)$

$3x^2dx=du$

$$\int 3x^2\cos(x^3)\,dx=\int\cos(u)\,du=\sin(u)+C=$$

$$=\sin(x^3)+C$$

(f) $\int te^{t^2+1}\,dt$

Method 1:

$t^2+1=u(t)$

$2t\,dt=du$

$t\,dt=\frac{du}{2}$

$$\int te^{t^2+1}dt=\int e^u\frac{du}{2}=\frac{e^u}{2}+C=$$

$$=\frac{e^{t^2+1}}{2}+C$$

Method 2:

$e^{t^2+1}=u(t)$

$2te^{t^2+1}dt=du$

$te^{t^2+1}dt=\frac{du}{2}$

$$\int te^{t^2+1}dt=\int\frac{du}{2}=\frac{1}{2}\int du=$$

$$=\frac{1}{2}u+C=\frac{1}{2}e^{t^2+1}+C.$$

(g) $\int e^{\cos x} \sin x \, dx$

$\cos x = u(x)$

$-\sin x dx = du \Rightarrow \sin x dx = -du$

$-\int e^u \, du = -e^u + C = -e^{\cos x} + C.$

$\int x\sqrt{2x-1} \, dx$ (Method 2)

Integration by parts:

$x = u \qquad \sqrt{2x-1}\, dx = dv = v' dx$

$dx = du \qquad \sqrt{2x-1} = v'$

$v = \int \sqrt{2x-1}\, dx$ = now substitution method.

$y(x) = 2x-1$

$dy = 2dx \Rightarrow dx = \frac{dy}{2}$

$v = \int y^{\frac{1}{2}} \frac{dy}{2} = \frac{1}{2}\int y^{\frac{1}{2}} dy = \frac{1}{2} \frac{y^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{3} y^{\frac{3}{2}} + C =$

$= \frac{1}{3}(2x-1)^{\frac{3}{2}} + C.$

One can shown that the two answers obtained in Method 1 and method 2 are equal to each other!

back to integration by parts:

$\int x\sqrt{2x-1}\, dx = x \cdot \frac{1}{3}(2x-1)^{\frac{3}{2}} - \int \frac{1}{3}(2x-1)^{\frac{3}{2}} dx =$

$= \frac{x}{3}(2x-1)^{\frac{3}{2}} - \frac{1}{3}\int (2x-1)^{\frac{3}{2}} dx$ [for the integral use the substitution $y(x) = 2x-1$; $dx = \frac{dy}{2}$]

$= \frac{x}{3}(2x-1)^{\frac{3}{2}} - \frac{1}{3}\int y^{\frac{3}{2}} \frac{dy}{2} = \frac{x}{3}(2x-1)^{\frac{3}{2}} - \frac{1}{6} \frac{y^{\frac{5}{2}}}{\frac{5}{2}} + C =$

$= \frac{x}{3}(2x-1)^{\frac{3}{2}} - \frac{1}{15}(2x-1)^{\frac{5}{2}} + C$