York University
Department of Mathematics and Statistics
Math 1505: Mathematics for Life and Social Sciences
Midterm Examination Dec 15, 2010

Name: ______________________________
Last Given

Student Number: ______________________

- **Answer all questions**
- **All work must be shown**
- **Total points 100**
- **Nongraphing, nonprogrammable calculator permissible**

Instructors:
Pietrowski: Sec A, MWF @ 10:30 (CLH E)
Pietrowski: Sec B, MWF @ 9:30 (CLH A)
Purzitsky : Sec C, MWF @ 9:30 (CLH E)
Chan : Sec D, Tues @ 7:00 (CLH A)
Purzitsky : Sec E, MWF @ 10:30 (CLH G)
Brettler : Sec G, MWF @ 10:30 (CLH F)

Print in the box, in bold letter, your section

ANSWERS

Question	Pts	Score
1	6	
2	5	
3	24	
4	5	
5	24	
6	5	
7	5	
8	5	
9	5	
10	5	
11	5	
12	6	
Total	100	

1.[6 points] Solve the system of linear equations

$$\begin{cases} x+y-2z=-7 \\ 3x-5y+2z=11 \\ x-2y+2z=10 \end{cases}$$

$$x+y-2z=-7$$
$$-8y+8z=32$$
$$-3y+4z=17$$

$$x+y-2z=-7$$
$$y-z=-4$$
$$-3y+4z=17$$

$$x+y-2z=-7$$
$$y-z=-4$$
$$z=5$$

If $z=5$, $y-5=-4$ gives $y=1$

If $y=1$, $z=5$, $x+1-10=-7$ gives $x=2$

$$\boxed{\text{ANSWER: } x=2, y=1, z=5}$$

2[5 points] Let $f(x) = x^2-2$. The bisection method is used to find an approximate value of x that will satisfy the equation $f(x) = 0$. Find the next three lines of the following table.

a	$\frac{a+b}{2}$	b	f(a)	$f\left(\frac{a+b}{2}\right)$	f(b)
1	1.25	1.5	-1	-0.4375	0.25
1,25	1,375	1,5	-0,4375	-0,1093	0,25
1,375	1,4375	1,5	-0,1093	.0664	0,25
1,375	1,40625	1,4375	-0,1093	-0,2246	0,0664

3.[24 points] Compute the following limits:

a) $\lim\limits_{x \to 2} \frac{x^2-4}{2x^2-x-6}$

$$\lim_{x \to 2} \frac{(x+2)(x-2)}{(2x+3)(x-2)} = \lim_{x \to 2} \frac{x+2}{2x+3} = \boxed{\frac{4}{7}}$$

b) $\lim\limits_{x \to \frac{15}{7}} x \cdot f(x+1)$ where $f(x) = \lfloor x \rfloor$ is the floor function.

$$\lim_{x \to \frac{15}{7}} x \lim_{x \to \frac{15}{7}} \lfloor x+1 \rfloor = \frac{15}{7} \cdot \left\lfloor \frac{22}{7} \right\rfloor$$

$$= \frac{15}{7} \cdot 3 = \boxed{\frac{45}{7}}$$

c) $\lim\limits_{x\to\infty}\dfrac{e^{x}+5e^{-x}}{7e^{x}-11e^{-x}} = \lim\limits_{x\to\infty}\dfrac{e^{x}+5e^{-x}}{7e^{-x}-11e^{-x}}\cdot\dfrac{e^{-y}}{e^{-x}}$

$= \lim\limits_{x\to\infty}\dfrac{1+5e^{-2x}}{7-11e^{-2x}} = \boxed{\dfrac{1}{7}}$

d) $\lim\limits_{x\to 0}\dfrac{\sqrt{3-x}-\sqrt{3}}{x} = \lim\limits_{x\to 0}\dfrac{\sqrt{3-x}-\sqrt{3}}{x}\cdot\dfrac{\sqrt{3-x}+\sqrt{3}}{\sqrt{3-x}+\sqrt{3}}$

$= \lim\limits_{x\to 0}\dfrac{-x}{x(\sqrt{3-x}+\sqrt{3})} = \lim\limits_{x\to 0}-\dfrac{1}{\sqrt{3-x}+\sqrt{3}}$

$= \boxed{-\dfrac{1}{2\sqrt{3}}}$

e) $\lim\limits_{x\to\infty}\dfrac{\frac{3}{x^2}-\frac{1}{x}}{\frac{2}{x^2}+\frac{7}{x}} = \lim\limits_{x\to\infty}\dfrac{\frac{3}{x^2}-\frac{1}{x}}{\frac{2}{x^2}+\frac{7}{x}}\cdot\dfrac{x}{x}$

$= \lim\limits_{x\to\infty}\dfrac{\frac{3}{x}-1}{\frac{2}{x}+7} = \boxed{-\dfrac{1}{7}}$

f) $\lim_{x\to 0} \frac{\sin(5x)}{\sin(3x)} = \lim_{x\to 0} \frac{\sin(5x)}{5x} \frac{3x}{\sin(3x)} \frac{5}{3}$

$= \boxed{\frac{5}{3}}$

4[5 points] Let $f(x) = \begin{cases} \frac{x^3+x-2}{x-1}, & x \neq 1 \\ 3, & x = 1 \end{cases}$

a) Find $\lim_{x\to 1} f(x)$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^3+x-2}{x-1} = \lim_{x\to 1} \frac{(x^2+x+2)(x-1)}{x-1}$

$= \lim_{x\to 1} x^2+x+2 = 4$

b) Is f(x) continuous at x=1? Justify your answer.

No as $\lim_{x\to 1} x^2+x+2 \neq f(1)$.

d) $f(t) = t^2 e^{\sqrt{t}}$

$$f'(t) = 2te^{\sqrt{t}} + t^2 e^{\sqrt{t}} \frac{1}{2\sqrt{t}}$$

OR

$$2te^{\sqrt{t}} + t^2 e^{\sqrt{t}} \left(\frac{1}{2} t^{-\frac{1}{2}}\right)$$

e) $f(x) = \arcsin\left(\dfrac{1}{1+x^2}\right)$

$$f'(x) = \frac{1}{\sqrt{1 - \left(\frac{1}{1+x^2}\right)^2}} \left(\frac{-2x}{(1+x^2)^2}\right)$$

f) $f(t) = \cos^2\left(\dfrac{1}{t^3}\right)$

$$f'(t) = 2\cos\left(\frac{1}{t^3}\right)\left(-\sin\left(\frac{1}{t^3}\right)\right)\left(-3t^{-4}\right)$$

5[24 points] Find the derivative of the following functions and do **not** simplify your answer:

a) $f(t)=\frac{5}{t}+\frac{3}{\sqrt{t}}+\ln(2\pi)$

$$f'(t) = -\frac{5}{t^2} - \frac{3}{2}t^{-3/2}$$

OR

$$= -5t^{-2} - \frac{3}{2}t^{-3/2}$$

b) $f(x)=\left(\frac{2x^2-1}{3x^2+1}\right)^4$

$$f'(x) = 4\left(\frac{2x^2-1}{3x^2+1}\right)^3 \left[\frac{(4x)(3x^2+1) - (2x^2-1)(6x)}{(3x^2+1)^2}\right]$$

c) $f(x)=\ln\left(\frac{x^3+2x}{x^2-1}\right)$

$$f(x) = \ln(x^3+2x) - \ln(x^2-1)$$

$$f'(x) = \frac{1}{x^3+2x}(3x^2) - \frac{1}{x^2-1}(2x)$$

8[5 points] Using the **formal definition** of derivative, find the derivative of $f(x) = 1 - \frac{1}{x}$.

$$\lim_{h \to 0} \frac{\left(1 - \frac{1}{x+h}\right) - \left(1 - \frac{1}{x}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x} - \frac{1}{x+h}}{h}$$

$$= \lim_{h \to 0} \frac{\left(\frac{h}{x(x+h)}\right)}{h}$$

$$= \lim_{h \to 0} \frac{1}{x(x+h)}$$

$$= \frac{1}{x^2}$$

6.[5 points]Find an equation of the tangent line to the curve $y = f(x) = \sqrt{3x^4 + 1}$ at x=2.

$$\frac{dy}{dx} = \frac{1}{2\sqrt{3x^4+1}}(12x^3)$$

When $x = 2$, $\frac{dy}{dx} = \frac{1}{2 \cdot 7} 12 \cdot 8 = \frac{48}{7}$

When $x = 2$ $y = 7$

Equation is

$$y - 7 = \frac{48}{7}(x-2)$$

or equivalent

7[5 points] Find the derivative of function f(x)= $(x^2+1)^{\arctan x}$

$$\ln(f(x)) = \ln(x^2+1)^{\arctan x}$$

$$= \arctan x \ln(x^2+1)$$

$$\frac{1}{f(x)} f'(x) = \frac{1}{1+x^2} \ln(x^2+1) + (\arctan x)\frac{1}{x^2+1}(2x)$$

$$\boxed{f'(x) = (x^2+1)^{\arctan x}\left[\frac{1}{1+x^2}\ln(x^2+1) + (\arctan x)\frac{2x}{x^2+1}\right]}$$

OR equivalent

11.[5 points]Suppose $f(x) = \sqrt{5}x^2$. If x=30mm is accurate to within 2mm, how accurate is f(30) in mm?

Error is $f'(x)dx$

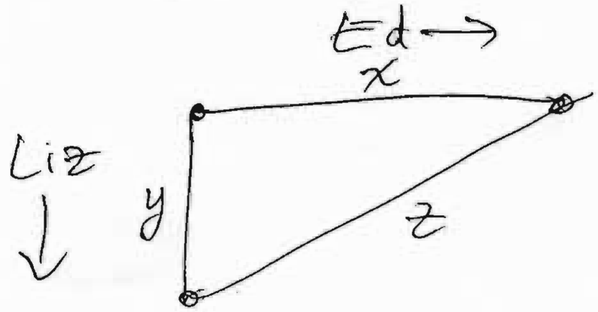
$f'(x) = 2\sqrt{5}x$

$x = 30 \quad dx = 2\text{mm}$

Error is $2\sqrt{5}x\,dx = 2\sqrt{5}(30)(2)$

Accurate to within $\boxed{120\sqrt{5}\text{ mm}}$

12.[6 points]] At 12 noon, Ed begins to walk east at the rate of 6 kilometers per hour. Two hours later, Liz at the same starting point, walks south at the rate of 5 kilometers per hour. What is the rate at which the distance between Ed and Liz is changing at 4 p.m.?



$z^2 = x^2 + y^2$; $\frac{dx}{dt} = 6$, $\frac{dy}{dx} = 5$

Find $\frac{dz}{dt}$

when $x = 24$, $y = 10$

[Note : When $x = 24, y = 10, z = 26$]

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$$

$$= \frac{1}{26}\left((24)(6) + (10)(5)\right)$$

$$= \boxed{\frac{194}{26}}$$ OR equivalent

9[5 points] Find equation of the tangent line to the curve $x^2+2xy^2-y^3=7$ at the point (2, 1). Express the equation in point-slope form

$$2x+(2)(y^2)+(2x)\left(2y\frac{dy}{dx}\right)-3y^2\frac{dy}{dx}=0$$

Set $x=2,\ y=1$

$$4+2+8\frac{dy}{dx}-3\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{6}{5}$$

$$\boxed{\text{Tangent has eqn } (y-1)=-\frac{6}{5}(x-2)}$$

10.[5 points] Let $f(x)=x^3+x+1$. Find $\frac{d}{dx}f^{-1}(x)$ at x=3.

$$f^{-1}(3)=1$$

$$f'(x)=3x^2+1$$

$$\frac{d}{dx}\left(f^{-1}(x)\right)=\frac{1}{f'(f^{-1}(x))}=\frac{1}{f'(f^{-1}(3))}$$

$$=\frac{1}{f'(1)}=\frac{1}{4}$$