

MAT 1332 Practice Midterm 1

1. Find the indefinite integral:

$$\int \left[\frac{1}{1-7x} + (x-1)^9 + \cos(9x) - \frac{5}{\sqrt{1-4x^2}} \right] dx.$$

2. Evaluate the definite integral $\int_0^{\frac{\pi}{2}} x^2 \sin x \, dx$.

3. Calculate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin x \, dx}{1 + \cos^2 x}$.

4. Evaluate $\int \frac{-2x+3}{x^2-x-2} dx$.

5. For each of the following improper integrals, determine whether it converges, and determine whether it converges, and determine its value if it does.

(a) $\int_1^3 \frac{dt}{t \ln t}$

(b) $\int_0^{\infty} \frac{3dt}{18+2t^2}$

(c) $\int_0^{\infty} x e^{-x} dx$

6. Solve the differential equation

$$\frac{dy}{dt} = \frac{6t \sin t}{y}$$

with initial condition $y(0) = 5$.

7. Consider the separable differential equation $\frac{dx}{dt} = \frac{3x}{t^2}$ with the initial condition $x(1) = e^{-1}$. Evaluate $x(3)$.

8. Let $f(x) = \frac{x}{1+x}$, $g(x) = \frac{x}{2}$.

(a) Show that the graphs of the functions intersect at $x = 0$ and $x = 1$.

(b) Find the area enclosed by the two graphs between the two points of intersection.

9. What is the average value of the function $f(x) = x \cos(2x)$ on $[0, \pi]$.

10. Find the volume that results from rotating the graph of $f(x) = x^{\frac{1}{3}}$, $0 \leq x \leq 8$, around the x -axis.

11. According to the Newton's Law of Cooling, the temperature of a warm object decreases at a rate proportional to the difference between its temperature and that of its surroundings. Namely,

$$\frac{dH}{dt} = \alpha(H - A)$$

where $H(t)$ is the temperature of the object at time t , α is the proportionality constant, and A is the temperature of the surroundings. When a murder is committed, the body, originally at 37.6° C, cools according to Newton's Law. Suppose that after three hours the body cools down to 35° C, and that the temperature of the surrounding air is a constant 20° C.

Find the temperature $H(t)$ of the body as a function of time t (in hours) since the murder was committed.

12. Consider the differential equation

$$\frac{dx}{dt} = 25x - x^3.$$

- (a) Find all equilibria.
- (b) Determine the stability of each equilibrium point using the stability criteria or a test point approach.
- (c) Draw the phase-line diagram for the differential equation.
- (d) Graph the equilibrium solutions and some other solutions for $t > 0$. Indicate the behavior of the solution curves as $t \rightarrow \infty$.