MAT 1330, Fall 2011 Assignment 6

Due November 23 at the beginning of class.

Late assignments will **not** be accepted; **nor** will unstapled assignments.

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Student Name _____ Student Number _____

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature _____

QUESTION 1. Consider the function $f(x) = x^3 - 5x^2 + 6x$ of a single real variable x.

Show that there exist two distinct solutions of $f'(x) = 0$.

QUESTION 2.

a) Find the Taylor polynomial of degree n of $f(x) = e^x$ near $x = 1$.

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b) Approximate $\sqrt{e^3}$ using the Taylor polynomial obtained in (a) of degree $n = 3$.

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QUESTION 3. Complete the following steps needed to sketch the graph of the function

$$f(x) = e^{-\frac{x^2-3x+1}{2}}.$$

a) Determine the domain of $f(x)$:

Answer: $\square$

b) Determine the vertical asymptotes of $f(x)$:

$\square$

c) Determine the horizontal asymptote(s):

$\lim_{x\to+\infty} f(x) = \square$

$\lim_{x\to-\infty} f(x) = \square$

Answer: $y = \square$.

d) Determine the zeros of f (if any) and the value of f at $x = 0$:

Zeros of f: $\square$. $f(0) = \square$.

e) Compute the derivative $f'(x)$, find the critical points of $f(x)$, and determine the sign of $f'(x)$ and behavior of $f(x)$.

$f'(x) = \square$.

Critical point(s): $x = \square$.

x	$-\infty$	$\frac{3}{2}$	$+\infty$
sign of $f'(x)$		0	
behaviour of $f(x)$			

f) Compute the second derivative $f''(x)$, determine its sign, find the inflection points and determine the concavity of $f(x)$.

$f''(x) =$ [ ] .

x	$-\infty$	0.5	2.5	$+\infty$
sign of $f''(x)$		0	0	
concavity of $f(x)$				

Inflection points: $x_1 =$ [ ] and $x_2 =$ [ ].

g) Sketch the graph of $f(x)$.

[ ] .

QUESTION 4. Determine the following limits: (a) $\lim_{x\to 0} \dfrac{1-\cos(x)}{x\tan(x)}$

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(b) $\lim_{t\to 0} \dfrac{7^t - 6^t}{t}$

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(c) $\lim_{x\to 1} \dfrac{1-x+\ln x}{1+\cos(2\pi x)}$

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