1332 S12 hw4

July 3.

MAT 1332, Spring/Summer 2012 Assignment 4

Due ~~June 21~~ 2012 at the beginning of class.

Late assignments will **not** be accepted; **nor** will unstapled assignments.

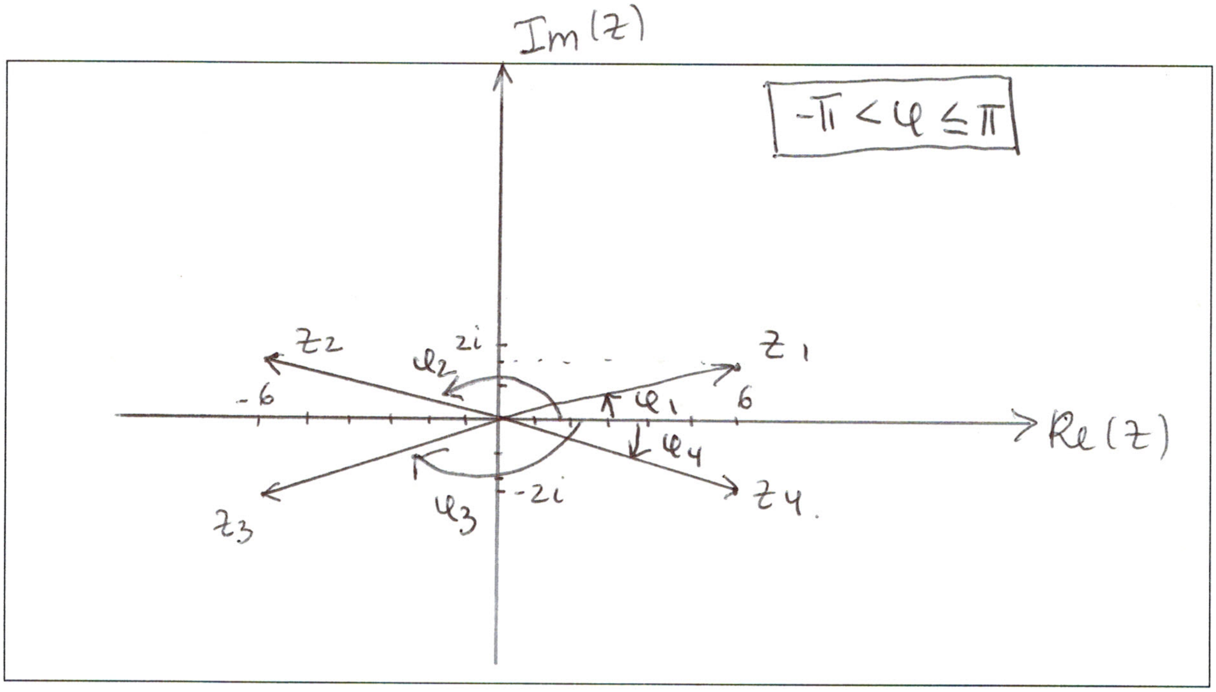
Instructor: Olga Vasilyeva

Student Name ______________________________ Student Number ______________________________

QUESTION 1. Consider the complex numbers

$$z_1 = 6 + \sqrt{3}i, \approx 6 + 1.7i$$
$$z_2 = -6 + \sqrt{3}i, \approx -6 + 1.7i$$
$$z_3 = -6 - \sqrt{3}i, \approx -6 - 1.7i$$
$$z_4 = 6 - \sqrt{3}i. \approx 6 - 1.7i$$

(a) Graph z_1, z_2, z_3, z_4 on $\mathbb{C}$.



(b) Write down two pairs of complex conjugates.

z_1 and z_4. $z_4 = \bar{z}_1$ or $\bar{z}_4 = z_1$

z_2 and z_3 $z_2 = \bar{z}_3$ or $\bar{z}_2 = z_3$

(c) Find the inverse z_1 (hint: $z \cdot \bar{z} = |z|^2 \Rightarrow$

$$z = \frac{|z|^2}{\bar{z}} \Rightarrow \frac{1}{z} = z^{-1} = \frac{\bar{z}}{|z|^2})$$

$$z_1 = \frac{\bar{z}_1}{|z_1|^2} = \frac{6 - \sqrt{3}\,i}{36 + 3} = \frac{6}{39} - \frac{\sqrt{3}}{39} i$$

(d) Express z_1, z_2, z_3, z_4 in polar coordinates

$$z = r(\cos\varphi + i\sin\varphi)$$

$$|z_1| = |z_2| = |z_3| = |z_4| = r = \sqrt{36 + 3} = \sqrt{39}$$

$$\varphi_1 = \arctan\left(\frac{y_1}{x_1}\right) = \arctan\left(\frac{\sqrt{3}}{6}\right) \approx 0.2810 \in \text{1-st quadr.}$$

$$\rightarrow z_1 = \sqrt{39}\left(\cos(0.281) + i\sin(0.281)\right)$$

$$\varphi_2 = \arctan\left(\frac{y_2}{x_2}\right) + \pi = \arctan\left(-\frac{\sqrt{3}}{6}\right) + \pi \approx 2.851 \in \text{2-nd}$$

$$\rightarrow z_2 = \sqrt{39}\left(\cos(2.851) + i\sin(2.851)\right)$$

$$\varphi_3 = \arctan\left(\frac{y_3}{x_3}\right) - \pi = \arctan\left(\frac{\sqrt{3}}{6}\right) - \pi \approx -2.8513 \in \text{3-d}$$

$$\rightarrow z_3 = \sqrt{39}\left(\cos(-2.8513) + i\sin(-2.8513)\right)$$

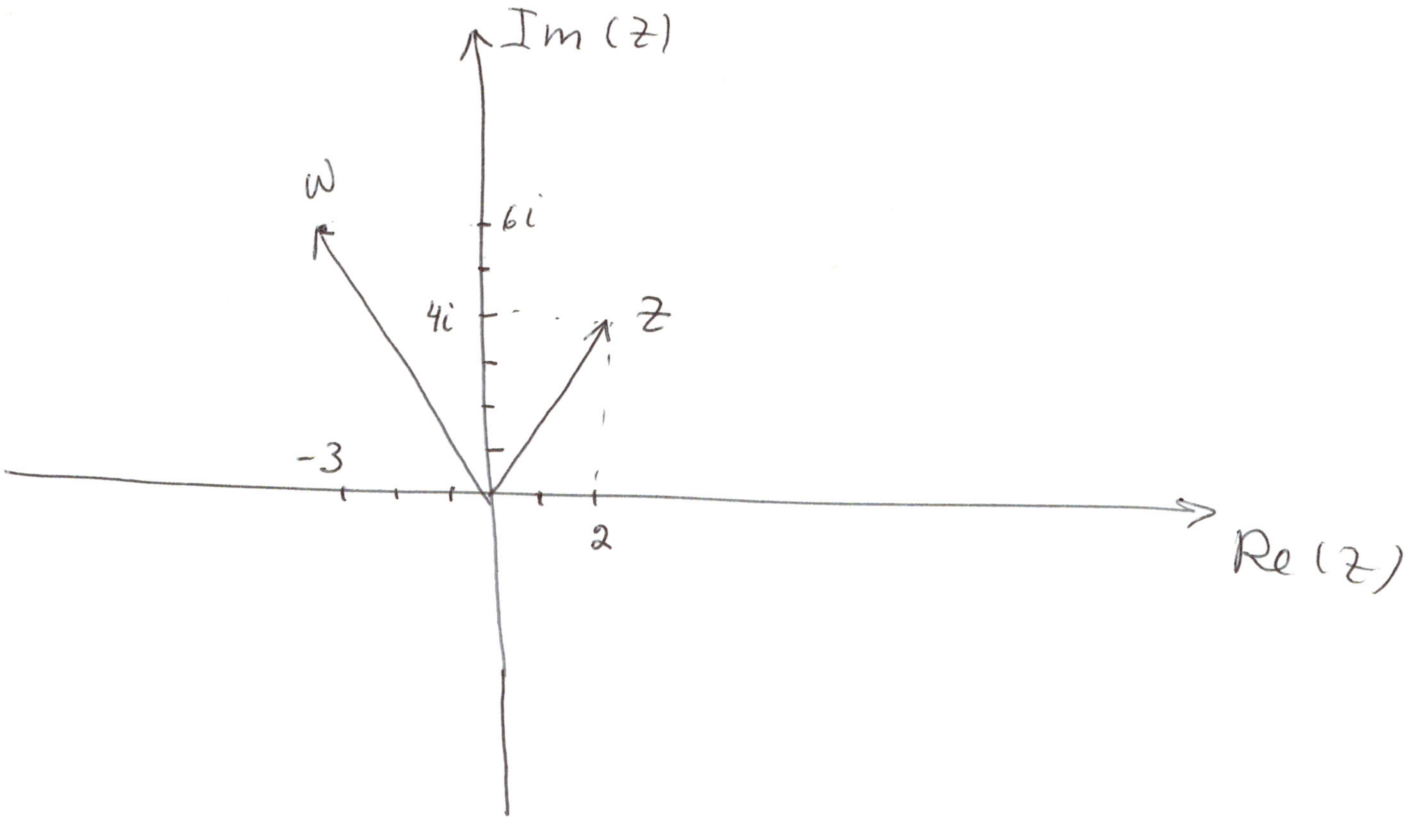
$$\varphi_4 = \arctan\left(\frac{y_4}{x_4}\right) = \arctan\left(-\frac{\sqrt{3}}{6}\right) \approx -0.2810$$

$$\rightarrow z_4 = \sqrt{39}\left(\cos(-0{,}2810) + i\sin(-0{,}2810)\right)$$

$\boxed{Q2}$ $z = 2+4i$

$w = -3+6i$

(a) Graph z, w in $\mathbb{C}$



(b) Find $z+w = 2+4i + (-3+6i) = -1+10i$

$z-w = 2+4i - (-3+6i) = 5-2i$

$i \cdot w = i(-3+6i) = -6-3i$

$z \cdot w = (2+4i)(-3+6i) = -6+12i-12i-24 = -30$

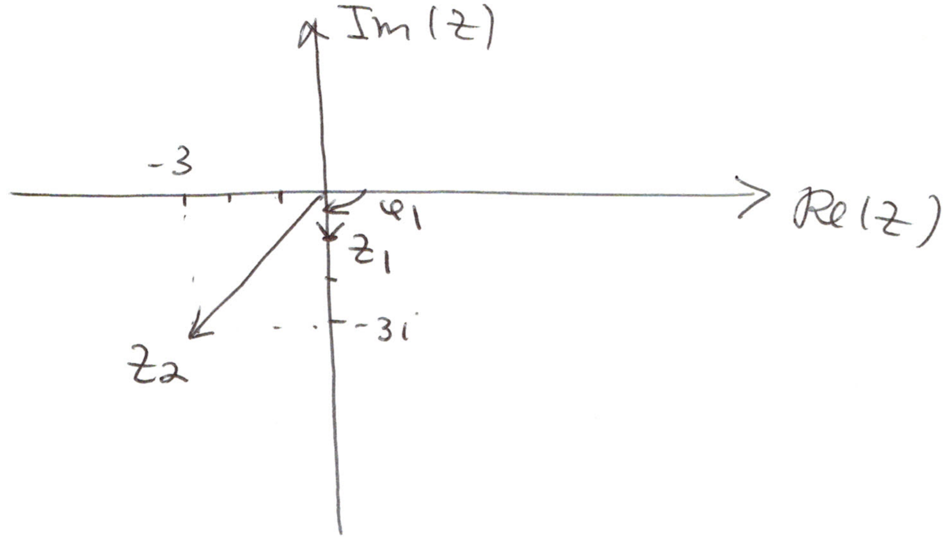
$$\frac{z}{w} = \frac{2+4i}{-3+6i} = \frac{(2+4i)(-3-6i)}{(-3+6i)(-3-6i)} =$$

$$\frac{-6-12i-12i+24}{(-3)^2-(6)^2(i)^2} = \frac{18-24i}{9+36} = \frac{18}{45} - \frac{24i}{45} =$$

$$= \frac{2}{5} - \frac{8i}{15}$$

$\boxed{Q3}$ $z_1 = -i = 0 - 1 \cdot i$

$z_2 = -3 - 3i = x + yi$



$$|z_1| = r = \sqrt{(x_1)^2 + (y_1)^2} = \sqrt{0^2 + (-1)^2} = 1$$

$$\begin{matrix} x_1 = r\cos\varphi_1 \\ y_1 = r\sin\varphi_1 \end{matrix} \Rightarrow \begin{cases} \cos\varphi_1 = 0 \\ \sin\varphi_1 = -1 \end{cases} \Rightarrow \varphi_1 = -\frac{\pi}{2}.$$

$$z_1 = 1 \cdot \left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right) = \underbrace{\boxed{1 \cdot e^{-\frac{\pi}{2}i}}}_{\text{Euler's notation}}$$

$$|z_2| = r = \sqrt{(x_1)^2 + (y_1)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = \cancel{2\sqrt{3}}$$

$$3\sqrt{2}$$

$$z_2 = 3\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right), \text{ where}$$

$$\varphi_2 = \arctan\left(\frac{y_2}{x_2}\right) - \pi = \arctan(1) - \pi =$$

$$= \frac{\pi}{4} - \pi = -\frac{3\pi}{4}; \quad \boxed{z_2 = 3\sqrt{2} \cdot e^{-\frac{3\pi}{4}i}}$$

(c) Write z_1, z_2 using the Euler notation.

$z_1 = 1 \cdot e^{-i \cdot \frac{\pi}{2}}$
$z_2 = 3\sqrt{2}\, e^{-\frac{3\pi}{4} i}$

(d) Find $z_1^3 = r^3(\cos(3\phi) + i\sin(3\phi))$
(the general formula states $z_1^n = r^n(\cos(n\phi) + i\sin(n\phi))$).

$z_1 = 1\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$
$z_1^3 = 1^3\left(\cos\left(-\frac{3\pi}{2}\right) + i\sin\left(-\frac{3\pi}{2}\right)\right) = +i$
Indeed, $z_1 = -i$, $z_1^3 = (-i)(-i)(-i) = -1 \cdot \underbrace{i \cdot i}_{-1} \cdot i = i$

QUESTION 4. For the system of linear equations

$$
\begin{aligned}
x + 3y + 9z &= 3 \\
2x + 7y + 23z &= 2 \\
x + ay + a^2 z &= a.
\end{aligned}
$$

(a) Determine the values of a for which the system has
 (i) no solution
 (ii) infinitely many solutions
 (iii) a unique solution.

$\left[\begin{array}{ccc|c} 1 & 3 & 9 & 3 \\ 2 & 7 & 23 & 2 \\ 1 & a & a^2 & a \end{array}\right]$ $\begin{array}{l} R1 \to R1 \\ R2 \to R2 - 2R1 \\ R3 \to R3 - R1 \end{array}$ $\left[\begin{array}{ccc|c} 1 & 3 & 9 & 3 \\ 0 & 1 & 5 & -4 \\ 0 & a-3 & a^2-9 & a-3 \end{array}\right]$ $\begin{array}{l} R1 \to R1 \\ R2 \to R2 \\ R3 \to R3 - (a-3)R2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 3 & 9 & 3 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & a^2-5a+6 & 5a-15 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 9 & 3 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & (a-3)(a-2) & 5(a-3) \end{array}\right]$

(i) if $a-2=0$ and $a-3\neq 0$, we have

$$\left[\begin{array}{ccc|c} 1 & 3 & 9 & 3 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 0 & 5(a-3) \end{array}\right] \to \text{no solution}$$

$$\xrightarrow{\text{or}} \left[\begin{array}{ccc|c} 1 & 3 & 9 & 3 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 0 & -5 \end{array}\right]$$

(ii) if $a-3=0$ $(a=3)$, we have $\left[\begin{array}{ccc|c} 1 & 3 & 9 & 3 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right]$

infinitely many solutions

(iii) $a-2\neq 0$ $(a\neq 2)$ and $a-3\neq 0$ $(a\neq 3)$

we have a unique solution

(b) In case (ii) above describe all solutions.

$x_3 = t,\ t \in \mathbb{R}$

$$\left[\begin{array}{ccc|c} 1 & 3 & 9 & 3 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -6 & 15 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

reduced row echelon form

$x_2 = -4-5x_3 = -4-5t$

$x_1 = 15+6x_3 = 15+6t$

$(x_1, x_2, x_3) = \{(15+6t, -4-5t, t), t \in \mathbb{R}\}$

(c) If $a = 1$, find the inverse matrix and find the solution to $A\bar{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 9 & 1 & 0 & 0 \\ 2 & 7 & 23 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \longrightarrow \begin{array}{l} R1 \to R1 \\ R2 \to R2 - 2R1 \\ R3 \to R3 - R1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 9 & 1 & 0 & 0 \\ 0 & 1 & 5 & -2 & 1 & 0 \\ 0 & -2 & -8 & -1 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R1 \to R1 \\ R2 \to R2 \\ R3 \to R3 + 2R2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 9 & 1 & 0 & 0 \\ 0 & 1 & 5 & -2 & 1 & 0 \\ 0 & 0 & 2 & -5 & 2 & 1 \end{array}\right] \quad R3 \to \frac{R3}{2} \left[\begin{array}{ccc|ccc} 1 & 3 & 9 & 1 & 0 & 0 \\ 0 & 1 & 5 & -2 & 1 & 0 \\ 0 & 0 & 1 & -\frac{5}{2} & 1 & \frac{1}{2} \end{array}\right]$$

$$\begin{array}{l} R1 \to R1 - 9R3 \\ R2 \to R2 - 5R3 \\ R3 \to R3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & +\frac{47}{2} & -9 & -\frac{9}{2} \\ 0 & 1 & 0 & \frac{21}{2} & -4 & -\frac{5}{2} \\ 0 & 0 & 1 & -\frac{5}{2} & 1 & \frac{1}{2} \end{array}\right] \quad R1 \to R1 - 3R2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -8 & 3 & 3 \\ 0 & 1 & 0 & \frac{21}{2} & -4 & -\frac{5}{2} \\ 0 & 0 & 1 & -\frac{5}{2} & 1 & \frac{1}{2} \end{array}\right]$$

$$\bar{x} = A^{-1}\bar{b} = \begin{bmatrix} -8 & 3 & 3 \\ \frac{21}{2} & -4 & -\frac{5}{2} \\ -\frac{5}{2} & 1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ -1 \end{bmatrix}$$

QUESTION 5. Consider the following matrices and vectors:

$$A = \begin{pmatrix} 1 & -7 & -9 \\ 6 & 0 & 8 \\ 4 & 4 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -7 \\ 6 & 4 \\ 0 & 1 \end{pmatrix}, \quad \bar{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix},$$

$$\bar{v} = \begin{pmatrix} 1 \\ \frac{7}{4} \\ 0 \end{pmatrix}, \quad \bar{w} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{3}{4} \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 3 & 0 & 1 \end{pmatrix}.$$

Compute the following if possible. If not possible, explain in one sentence why.

(a) $3\bar{v} + 2B^T\bar{w}$

(b) $\bar{w}\bar{v}$

(c) $\bar{v}^T\bar{w}$

(d) $A^TB + 2\bar{v}^T\bar{w}$

(e) AB

(f) $B\bar{u}$

(g) BA

(h) D^2, $\det(D)$

(j) $A + D$

(a) is not defined B^T is (2×3)

$B^T \cdot w = (2 \times 3)\cdot(3\times 1) = (2\times 1)$

$3\bar{v}$ is (3×1). can add $3\bar{v}$ and $B^T w$

(b) $\bar{w}\bar{v}^T = (3\times 1)(1\times 3) = (3\times 3)$

$$\bar{w}\bar{v}^T = \begin{bmatrix} 0 \\ \frac{1}{2} \\ \frac{3}{4} \end{bmatrix} \begin{bmatrix} 1 & \frac{7}{4} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{2} & \frac{7}{8} & 0 \\ \frac{3}{4} & \frac{21}{16} & 0 \end{bmatrix}$$

(c) $\bar{v}^T \cdot w = (1\times 3)(3\times 1) = (1\times 1)$

$$\begin{bmatrix} 1 & \frac{7}{4} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{2} \\ \frac{3}{4} \end{bmatrix} = 0\cdot 1 + \frac{7}{4}\cdot\frac{1}{2} + 0\cdot\frac{3}{4} = \frac{7}{8}$$

(d) $A^T \cdot B = (3 \times \underline{\underline{3}})(\underline{\underline{3}} \times 2) = (3 \times 2)$

$2 \cdot \bar{v}^T \cdot \bar{w} = \frac{7}{8} \cdot 2 = \cancel{14} \ \frac{7}{4}$

$A^T \cdot B + 2\bar{v}^T \bar{w}$ is not defined.

(e) $AB = (3 \times \underline{\underline{3}})(\underline{\underline{3}} \times 2) = (3 \times 2)$

$$\begin{bmatrix} 1 & -7 & -9 \\ 6 & 0 & 8 \\ 4 & 4 & 0 \end{bmatrix} \begin{bmatrix} 3 & -7 \\ 6 & 4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -39 & -44 \\ 18 & -34 \\ 42 & -12 \end{bmatrix}$$

(f) $B\bar{u} = (3 \times \underline{\underline{2}})(\underline{\underline{3}} \times 1)$ is not defined

(g) $BA = (3 \times \underline{\underline{2}})(\underline{\underline{3}} \times 3)$ is not defined

(h) $\cancel{AB} \ D^2 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} =$

$= \begin{bmatrix} 1 & 4 & 4 \\ 6 & 1 & 4 \\ 6 & 6 & 1 \end{bmatrix}$, $\det(D) = 1 \cdot \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 0 & 2 \\ 3 & 1 \end{vmatrix} =$

$= 1(1 - 2 \cdot 0) - 2(0 \cdot 1 - 3 \cdot 2) = 1 + 12 = 13$

(j) $A + D = \begin{bmatrix} 1 & -7 & -9 \\ 6 & 0 & 8 \\ 4 & 4 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -5 & -9 \\ 6 & 1 & 10 \\ 7 & 4 & 1 \end{bmatrix}$

QUESTION 6. Consider

$$
\begin{aligned}
x_1 + 2x_2 + 3x_3 &= 5 \\
3x_1 + 2x_2 + 4x_3 &= 4 \\
2x_1 + x_2 + 2x_3 &= 2.
\end{aligned}
$$

Reduce the corresponding augmented matrix to reduced row-echelon form.

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 3 & 2 & 4 & 4 \\ 2 & 1 & 2 & 2 \end{array}\right] \quad \begin{array}{l} R1 \to R1 \\ R2 \to R2 - 3R1 \\ R3 \to R3 - 2R1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & -4 & -5 & -11 \\ 0 & -3 & -4 & -8 \end{array}\right] \quad \begin{array}{l} R2 \to -\frac{\cancel{4}R2}{4} \\ R3 \to \frac{R3}{3} \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & \frac{5}{4} & \frac{11}{4} \\ 0 & -1 & -\frac{4}{3} & -\frac{8}{3} \end{array}\right] \quad R3 \to R3 + R2 \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & \frac{5}{4} & \frac{11}{4} \\ 0 & 0 & -\frac{1}{12} & \frac{1}{12} \end{array}\right] \quad R3 \to R3\cdot(-1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & \frac{5}{4} & \frac{11}{4} \\ 0 & 0 & 1 & -1 \end{array}\right] \quad \begin{array}{l} R1 \to R1 - 3R3 \\ R2 \to R2 - \frac{5}{4}R3 \\ R3 \to R3 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 2 & 0 & 8 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\begin{array}{l} R1 \to R1 - 2R2 \\ R2 \to R2 \\ R3 \to R3 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$X_3 = -1$

$X_2 = 4$

$X_1 = 0$

QUESTION 7. Given $A = \begin{pmatrix} 2 & 0 & 4 \\ 0 & -4 & 0 \\ 4 & 0 & 2 \end{pmatrix}$.

(a) Use the characteristic equation to find the eigenvalues.

$\det(A - \lambda I) = 0$

$\det \begin{bmatrix} 2-\lambda & 0 & 4 \\ 0 & -4-\lambda & 0 \\ 4 & 0 & 2-\lambda \end{bmatrix} = (2-\lambda)(-4-\lambda)(2-\lambda) - 4\cdot 4(-4-\lambda) = 0$

$(-4-\lambda)\left[(2-\lambda)(2-\lambda) - 16\right] = 0$

$-4-\lambda = 0$

$\lambda_1 = -4$

$4 - 4\lambda + \lambda^2 - 16 = 0$

$\lambda^2 - 4\lambda - 12 = 0$

$\mathcal{D} = 16 - 4(-12) = 64$

$\lambda_2 = \frac{4+8}{2} = 6$

$\lambda_3 = \frac{4-8}{2} = -2.$

(b) Find the corresponding eigenvectors.

$\boxed{\lambda_1 = -4}$

$$\begin{bmatrix} -2 & 0 & 4 \\ 0 & 0 & 0 \\ 4 & 0 & 6 \end{bmatrix} \begin{bmatrix} v_1^1 \\ v_2^1 \\ v_3^1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Augmented matrix

$$\left[\begin{array}{ccc|c} -2 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 0 & 6 & 0 \end{array}\right] \quad R1 \to -\frac{R1}{2} \quad R1 \leftrightarrow R3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 4 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R1 \to R1 \\ R2 \to R2 - 4R1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 0 & -14 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$R2 \to -\frac{R2}{14} \quad \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \cancel{v_3^1 \neq 0}. \quad R1 \to R1 + 2R2$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} v_3^1 = 0 \\ v_1^1 = 0 \end{array} \quad \begin{array}{l} v_2^1 = t, \ t \in \mathbb{R} \\ t \neq 0 \end{array}$$

$$\bar{v}^1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} t, \quad t \neq 0, \quad t \in \mathbb{R}.$$

$\boxed{\lambda_2 = 6}$

$$\begin{bmatrix} -4 & 0 & 4 \\ 0 & -10 & 0 \\ 4 & 0 & -4 \end{bmatrix} \begin{bmatrix} V_1^2 \\ V_2^2 \\ V_2^3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -4 & 0 & 4 & 0 \\ 0 & -10 & 0 & 0 \\ 4 & 0 & -4 & 0 \end{array}\right] \quad \begin{array}{l} R1 \to -\frac{R1}{4} \\ R2 \to -\frac{R2}{10} \\ R3 \to R3 + R1 \end{array} \quad \left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$V_2^2 = 0.$

$-V_1^2 + V_3^2 = 0$

$V_1^2 = V_3^2 = t, \quad t \neq 0, \quad t \in \mathbb{R}$

$$\bar{V}^2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} t, \quad t \neq 0, \quad t \in \mathbb{R}.$$

$\boxed{\lambda_3 = -2}$

$$\begin{bmatrix} 4 & 0 & 4 \\ 0 & -2 & 0 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1^3 \\ V_2^3 \\ V_3^3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 4 & 0 & 4 & 0 \\ 0 & -2 & 0 & 0 \\ 4 & 0 & 4 & 0 \end{array}\right] \quad \begin{array}{l} R1 \to \frac{R1}{4} \\ R2 \to -\frac{R2}{2} \\ R3 \to R3 - R1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$V_2^3 = 0$

$V_1^3 + V_3^3 = 0$

$V_1^3 = -V_3^3$

$$\bar{V}^3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} t, \quad t \neq 0, \quad t \in \mathbb{R}$$

$\boxed{Q8}$ $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$\det(A - \lambda I) = 0$

$\det \begin{bmatrix} 1-\lambda & -1 \\ 2 & 3-\lambda \end{bmatrix} = (1-\lambda)(3-\lambda) + 2 = 0$

$3 - 3\lambda - \lambda + \lambda^2 + 2 = \lambda^2 - 4\lambda + 5 = 0$

$\mathcal{D} = 16 - 4 \cdot 5 = -4 < 0$

$\lambda_1 = \dfrac{4 + \sqrt{-4}}{2} = \dfrac{4 + \sqrt{4 \cdot (-1)}}{2} = \dfrac{4 + \sqrt{4} \cdot \sqrt{-1}}{2} = \dfrac{4 + 2i}{2} \Rightarrow$

$\boxed{\lambda_1 = 2 + i}$

$\lambda_2 = \dfrac{4 - \sqrt{-4}}{2} = 2 - i$

(b) Find the eigenvector corresponding to the eigenvalue from the first quadrant.

$$\begin{bmatrix} 1-(2+i) & -1 \\ 2 & 3-(2+i) \end{bmatrix}\begin{bmatrix} V_1^1 \\ V_2^1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -1-i & -1 & 0 \\ 2 & 1-i & 0 \end{array}\right] \quad \begin{array}{l} R1 \to R1 \cdot (-1+i) \\ R2 \to R2 \end{array}$$

$$\left[\begin{array}{cc|c} (-1-i)(-1+i) & -1(-1+i) & 0 \\ 2 & 1-i & 0 \end{array}\right] \to$$

$$\left[\begin{array}{cc|c} 2 & 1-i & 0 \\ 2 & 1-i & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 2 & 1-i & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$V_2^1 = t,\ t \neq 0,\ t \in \mathbb{R}$

$2 \cdot V_1^1 + (1-i)V_2^1 = 0$

$$V_1^1 = -\frac{(1-i)V_2^1}{2} = \frac{(-1+i)V_2^1}{2} = \frac{(-1+i)\cdot t}{2}.$$

$$\bar{V}^1 = \begin{bmatrix} \frac{(-1+i)}{2} \\ 1 \end{bmatrix} t,\ t \neq 0,\ t \in \mathbb{R}.$$