Total: 10 points

MAT 1332
1332 S1hw1.
HW1

MAT 1332, Spring/Summer 2012 Assignment 1

Due May 17, 2012 at the beginning of class.

Late assignments will **not** be accepted; **nor** will unstapled assignments.

Instructor Olga Vasilyeva

Student Name ______________________ Student Number ______________________

QUESTION 1. Calculate

guess-and-check method or substitution method

(a) $\int \frac{1}{7t-2}dt$ $= \frac{1}{7}\int \frac{du}{u} = \frac{1}{7}\ln|u|+C = \frac{1}{7}\ln|7t-2|+C$

$7t-2 = u(t)$
$7 = \frac{du}{dt} \Rightarrow dt = \frac{du}{7}$

Check: $F(t) = \frac{1}{7}\ln|7t-2|$

$F(t) = \begin{cases} \frac{1}{7}\ln(7t-2) & \text{if } t \geq \frac{2}{7} \\ \frac{1}{7}\ln(2-7t) & \text{if } t < \frac{2}{7} \end{cases}$

$F'(t) = \frac{1}{7}\cdot 7 \cdot \frac{1}{7t-2} = \frac{1}{7t-2}$ if $t \geq 2/7$ and $F'(t) = \frac{1}{7}(-7)\frac{1}{2-7t} = \frac{1}{7t-2}$ if $t < \frac{2}{7}$.

(b) $\int \frac{1}{3-5t}dt$ $= -\frac{1}{5}\int \frac{du}{u} = -\frac{1}{5}\ln|u|+C = -\frac{1}{5}\ln|3-5t|+C$;

$3-5t = u(t)$
$-5 = \frac{du}{dt} \Rightarrow dt = -\frac{du}{5}$

Check: $F(t) = -\frac{1}{5}\ln|3-5t|$

$F(t) = \begin{cases} -\frac{1}{5}\ln(3-5t), & \text{if } t \leq \frac{3}{5} \\ -\frac{1}{5}\ln(5t-3) & \text{if } t > \frac{3}{5} \end{cases}$

differentiating $F(t)$, gives you $\frac{1}{3-5t}$.

(c) $\int_2^3 (3y^4 - y^5)dy$ $= 3\int_2^3 y^4 dy - \int_2^3 y^5 dy$

$= \frac{3y^5}{5}\Big|_2^3 - \frac{y^6}{6}\Big|_2^3 = \frac{3}{5}(3^5 - 2^5) - \frac{1}{6}(3^6 - 2^6)$

guess-and-check method or substitution

(d) $\int 6\sin(30x+40)dx$ $= 6\int \sin(30x+40)dx =$

$30x+40 = u(x)$

$30 = \frac{du}{dx} \Rightarrow dx = \frac{du}{30}$

$= \frac{6}{30}\int \sin(u)du = -\frac{1}{5}\cos(u) + C =$

$= -\frac{1}{5}\cos(30x+40) + C.$

(e) $\int_0^{\pi/6} x\cos(x)dx$; $\int u\,dv = uv - \int v\,du$

integration by parts.

$x = u$ $\quad$ $\cos(x)dx = dv = v'(x)dx$

$dx = du$ $\quad$ $v'(x) = \cos(x)$

$v(x) = \int \cos(x)dx = \sin x$

$\int_0^{\frac{\pi}{6}} x\cos(x)dx = x\cdot\sin(x)\Big|_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} \sin x\,dx =$

$= \frac{\pi}{6}\cdot\sin\left(\frac{\pi}{6}\right) - 0\cdot\sin 0 + \cos(x)\Big|_{x=0}^{x=\frac{\pi}{6}} =$

$= \frac{\pi}{6}\cdot\frac{1}{2} + \cos\left(\frac{\pi}{6}\right) - \cos(0) = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

(a), (b), (c), (d) 0.5 each.

(e) - 1 point

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

(f) $\int_1^2 x^4 \ln(x)dx$ $=$

integration by parts

$$\boxed{\begin{array}{ll} x^4 dx = dv = v'(x)dx & \ln x = u \\ x^4 = v'(x) & \frac{dx}{x} = du \\ \frac{x^5}{5} = v(x) & \end{array}}$$

$$= \frac{x^5}{5} \cdot \ln x \Big|_1^2 - \int_1^2 \frac{x^5}{5} \cdot \frac{dx}{x} = \frac{2^5}{5}\ln 2 - \frac{1^5}{5}\ln 1 -$$

$$-\frac{1}{5}\int_1^2 x^4 dx = \frac{2^5}{5}\ln 2 - \frac{1}{5} \cdot \frac{x^5}{5}\Big|_1^2 = \frac{32}{5}\ln 2 -$$

$$- \frac{1}{25}\left[2^5 - 1^5\right] = \frac{32}{5}\ln 2 + \frac{1}{25} - \frac{32}{25}$$

(g) $\int \frac{e^{2x}}{\cos^2(e^{2x})}dx$

Substitution

$g(x) = e^{2x}$ is the inner function

$f(g) = \frac{1}{\cos^2(g)}$ is the outer function.

$$\boxed{\begin{array}{l} u(x) = e^{2x} \\ \frac{du}{dx} = 2e^{2x} \quad \Rightarrow \quad e^{2x}dx = \frac{du}{2}. \end{array}}$$

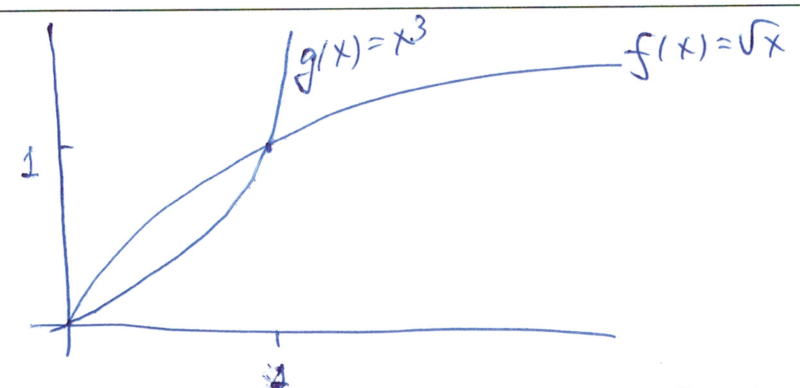
$$\int \frac{e^{2x}dx}{\cos^2(e^{2x})} = \frac{1}{2}\int \frac{du}{\cos^2(u)} = \frac{1}{2}\tan(u) + C =$$

$$= \frac{1}{2}\tan(e^{2x}) + C.$$

$\boxed{\text{(f), (g) 1 point each}}$

QUESTION 2. Find the area of the region between the graphs of functions

$f(x) = \sqrt{x}$ and $g(x) = x^3$. $\boxed{\text{2 points}}$



Points of intersection: $f(x) = g(x)$

$\sqrt{x} = x^3$

$x = x^6$

$x^6 - x = x(x^5 - 1) = 0$

$x_1 = 0 \quad ; \quad x_2 = 1$

$$\text{Area} = \int_0^1 |f(x) - g(x)|\,dx$$

Take a test point, for example

$\bar{x} = 0.5 \in (0,1)$

$f(\bar{x}) = f(0.5) = \sqrt{0.5} \approx 0.7$

$g(\bar{x}) = g(0.5) = 0.125$

$f(\bar{x}) > g(\bar{x})$ at any point $\bar{x} \in (0,1)$.

Thus, $\text{Area} = \int_0^1 (f(x) - g(x))\,dx = \int_0^1 (\sqrt{x} - x^3)\,dx =$

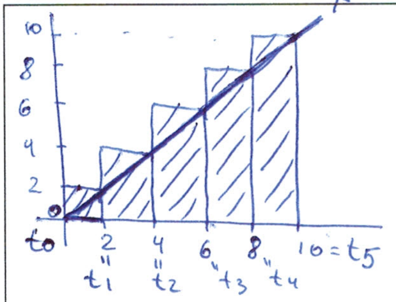
$$= \frac{2x^{\left(\frac{1}{2}+1\right)}}{3}\Bigg|_0^1 - \frac{x^4}{4}\Bigg|_0^1 = \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12}\,[\text{unit}]^2$$

QUESTION 3. Consider the function $f(x) = x$ on $[0, 10]$. Partition the interval $[0, 10]$ into five equal subintervals and evaluate

$\Delta t = \frac{b-a}{5} = \frac{10}{5} = 2$

(a) the right-hand Riemann sum, I_R (using the right endpoints);

$x = f(t)$



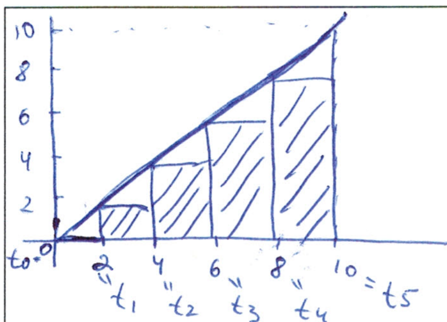
$I_R = \sum_{i=1}^{5} f(t_i)\,\Delta t =$

$= [f(t_1) + f(t_2) + f(t_3) + f(t_4) + f(t_5)] \cdot 2 =$

$= (2 + 4 + 6 + 8 + 10) \cdot 2 = 60$

1 point

(b) the left-hand Riemann sum, I_L (using the left endpoints);



$I_L = \sum_{i=0}^{4} f(t_i)\,\Delta t =$

$= [f(t_0) + f(t_1) + f(t_2) + f(t_3) + f(t_4)] \cdot 2 =$

$= [0 + 2 + 4 + 6 + 8] \cdot 2 = 40$

(c) $\int_0^{10} x\,dx$; $= \left.\frac{x^2}{2}\right|_{x=0}^{10} = \frac{100^2}{2} = 50.$

0.5 point

(d) Compare your answers in (a), (b) and (c). Which Riemann sum underestimates the actual value of $\int_0^{10} x dx$, and which Riemann sum overestimates it? Why? (short 1-2 sentence explanation is enough)

0.5 point

$$I_L \le \int_0^{10} x dx \le I_R$$

underestimates

overestimates.

$$40 \le 50 \le 60$$

It happened since the function f is an increasing function.

QUESTION 4. The region bounded by the curve $y = e^{-2x}$ and the x-axis between $x = 0$ and $x = 1$ is revolved around the x-axis. Find the volume of this solid of revolution.

1 point

$$V = \pi \int_0^1 (e^{-2x})^2 dx = \pi \int_0^1 e^{-4x} dx =$$

$$= \left. \frac{\pi e^{-4x}}{-4} \right|_{x=0}^{x=1} = -\frac{\pi}{4} e^{-4} + \frac{\pi}{4} e^0 =$$

$$= \left(\frac{\pi}{4} - \frac{\pi}{4e^4} \right) [\text{unit}]^3.$$

QUESTION 5. Suppose a flower grows in height according to the equation

$$\frac{dH}{dt} = 5e^{-0.2t},$$

$$H(0) = 6,$$

where $H(t)$ is the height of the flower (in cm) at time t (weeks).
(a) How much does the flower grow in the first 4 weeks (between $t = 0$ and $t = 4$)?

(a) $H(4) - H(0) = \int_0^4 H'(t)dt \overset{FTC}{=} \int H'(t)dt \Big|_{t=0}^{t=4} =$

$= \int 5e^{-0.2t} dt \Big|_0^4 = \frac{5e^{-0.2t}}{-0.2} \Big|_0^4 =$

$= -25(e^{-0.8} - e^0) = 25 - \frac{25}{e^{0.8}}$

(b) Is the growth in the long term (until $t = \infty$) finite or infinite?

$H(t) = \int 5e^{-0.2t} dt = \frac{5e^{-0.2t}}{-0.2} + C =$

$= -25e^{-0.2t} + C.$

$H(0) = -25 + C = 6 \quad \Rightarrow C = 31$

$H(t) = -25e^{-0.2t} + 31$

$H(t) \to 31$ (cm) as $t \to \infty$ (in the long term)
(finite)

(b) no, it will never reach 35 (cm).

(no points for this problem).