



(e) The second derivative of  $f$  is  $f'' =$

$$f''(x) = \frac{x^3(-1) - (8-x)3x^2}{x^6} = \frac{2(x-12)}{x^4}$$

(f) The point(s) of inflection are

Since  $f''(x) < 0$  for  $x < 12$  and  $f''(x) > 0$  for  $x > 12 \Rightarrow 12$  is the only inflection point

(g) Find  $\lim_{x \rightarrow 0^+} f(x) =$

$$-\infty$$

,  $\lim_{x \rightarrow 0^-} f(x) =$

$$-\infty$$

(h) Find  $\lim_{x \rightarrow -\infty} f(x) =$

$$0$$

,  $\lim_{x \rightarrow \infty} f(x) =$

$$0$$

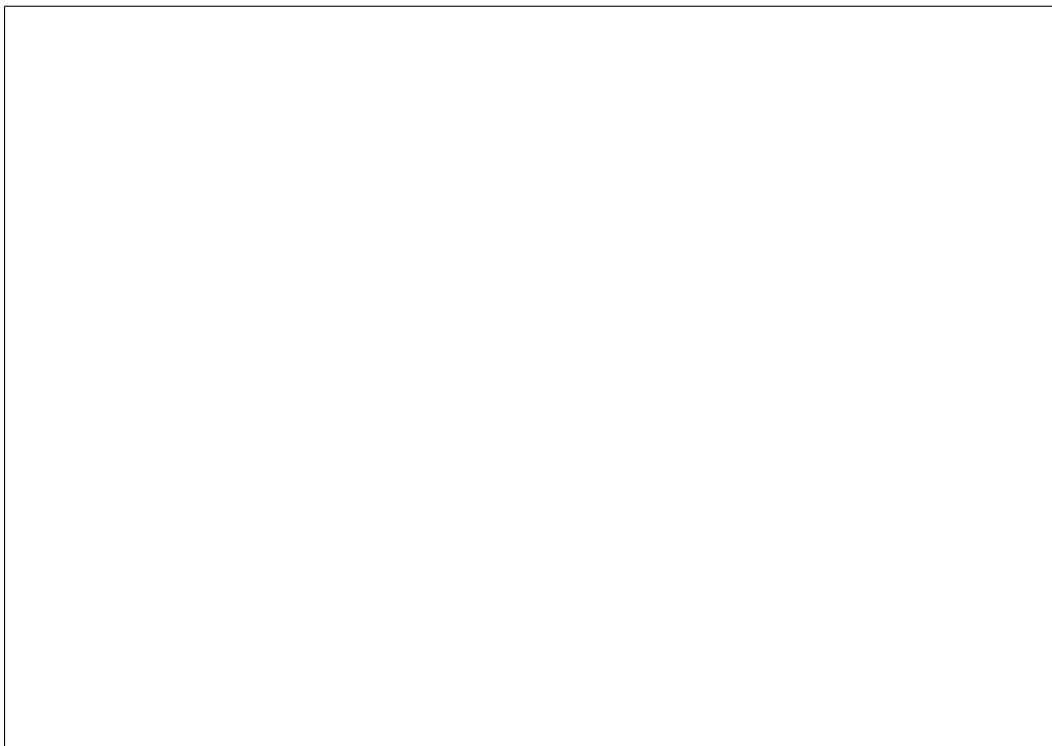
(i) The equation(s) of the vertical asymptote(s)

$$x = 0$$

(j) The equation(s) of horizontal asymptote(s)

$$y = 0$$

(k) The graph of  $f$  is



QUESTION 2. Consider the function  $f(x) = \frac{x}{2} + \cos(x)$ . Follow these steps to graph the function over the interval  $[0, \pi]$ .

(a) The derivative of  $f$  is  $f' =$

$$f'(x) = \frac{1}{2} - \sin(x)$$

(b) The critical point(s) of  $f$  are

$$f'(x) = 0 \Rightarrow \sin(x) = 0.5 \Rightarrow x = \frac{\pi}{6} \text{ or } x = \frac{5\pi}{6}$$

(c) The second derivative of  $f$  is  $f'' =$

$$f''(x) = -\cos(x)$$

(d) The point(s) of inflection are

Since  $f''(x) < 0$  for  $0 < x < \frac{\pi}{2}$  and  $f''(x) > 0$  for  $\pi > x > \frac{\pi}{2} \Rightarrow \frac{\pi}{2}$  is the only inflection point

(e) Find  $f(0) =$

1

,  $f(\frac{\pi}{6}) =$

1.13

Find  $f(\frac{\pi}{2}) =$

0.79

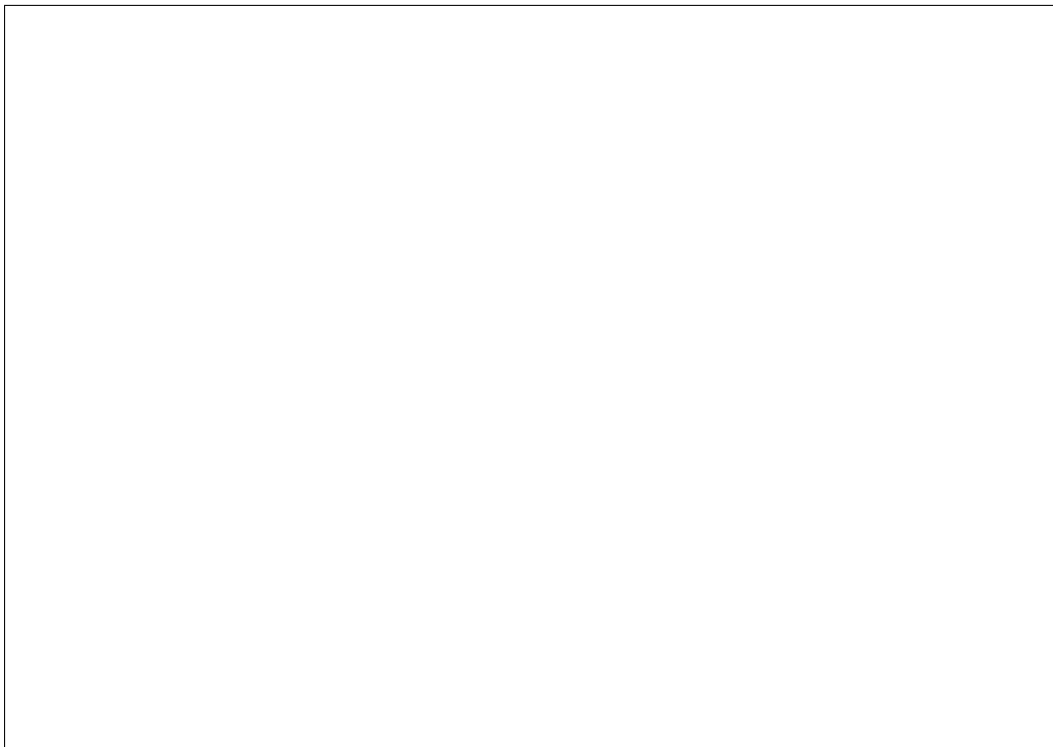
,  $f(\frac{5\pi}{6}) =$

0.44

Find  $f(\pi) =$

0.57

(f) The graph of  $f$  is



QUESTION 3. Find all local maximums and minimums (if any) and all inflection points of the function  $f(x) = xe^{2011x}$ .

Note that  $f'(x) = e^{2011x} + xe^{2011x}2011 = e^{2011x}(1 + 2011x)$ . Note that  $f'(x) = 0$  if and only if  $x = \frac{-1}{2011}$ . Since  $f'(x) < 0$  for  $x < \frac{-1}{2011}$  and  $f'(x) > 0$  for  $x > \frac{-1}{2011}$ , it follows that  $x = \frac{-1}{2011}$  is a local minimum.

Using again the product rule one may obtain that  $f''(x) = 2011e^{2011x} + (1+2011x)e^{2011x}2011 = e^{2011x}2011(2 + 2011x)$ , hence  $f''(x) = 0$  if and only if  $x = \frac{-2}{2011}$ . Since  $f''(x) < 0$  for  $x < \frac{-2}{2011}$  and  $f''(x) > 0$  for  $x > \frac{-2}{2011}$ , it follows that  $x = \frac{-2}{2011}$  is an inflection point.

Space for the last question.