MAT 1330, Fall 2011 Assignment 5

Due WED NOVEMBER 16, 8:30am at the beginning of class.

Late assignments will **not** be accepted; **nor** will unstapled assignments.

Instructor (circle one): Jason Levy   Robert Smith?   Olga Vassilieva   Catalin Rada

DGD (circle one): 1   2   3   4

Student Name ______________________ Student Number ______________________

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature ______________________

QUESTION 1. Consider the function $f(x) = \frac{x-4}{x^2}$. Follow these steps to graph the function.

(a) The domain of f is

$x \neq 0$

(b) The x-intercept(s) of f are

$f(x) = 0 \Rightarrow x - 4 = 0 \Rightarrow x = 4$

(c) The derivative of f is $f' =$

$f'(x) = \frac{x^2(1)-(x-4)(2x)}{x^4} = \frac{x(8-x)}{x^4} = \frac{8-x}{x^3}$

(d) The critical point(s) of f are

$f'(x) = 0 \Rightarrow 8 - x = 0 \Rightarrow x = 8$

(e) The second derivative of f is $f'' =$ $f''(x) = \frac{x^3(-1)-(8-x)3x^2}{x^6} = \frac{2(x-12)}{x^4}$

(f) The point(s) of inflection are

Since $f''(x) < 0$ for $x < 12$ and $f''(x) > 0$ for $x > 12 \Rightarrow 12$ is the only inflection point

(g) Find $\lim_{x\to 0^+} f(x) =$ $-\infty$, $\lim_{x\to 0^-} f(x) =$ $-\infty$

(h) Find $\lim_{x\to -\infty} f(x) =$ 0, $\lim_{x\to \infty} f(x) =$ 0

(i) The equation(s) of the vertical asymptote(s) $x = 0$

(j) The equation(s) of horizontal asymptote(s) $y = 0$

(k) The graph of f is

QUESTION 2. Consider the function $f(x) = \frac{x}{2} + \cos(x)$. Follow these steps to graph the function over the interval $[0, \pi]$.

(a) The derivative of f is $f' =$ $f'(x) = \frac{1}{2} - \sin(x)$

(b) The critical point(s) of f are $f'(x) = 0 \Rightarrow \sin(x) = 0.5 \Rightarrow x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$

(c) The second derivative of f is $f'' =$ $f''(x) = -\cos(x)$

(d) The point(s) of inflection are

Since $f''(x) < 0$ for $0 < x < \frac{\pi}{2}$ and $f''(x) > 0$ for $\pi > x > \frac{\pi}{2} \Rightarrow \frac{\pi}{2}$ is the only inflection point

(e) Find $f(0) =$ 1, $f(\frac{\pi}{6}) =$ 1.13

Find $f(\frac{\pi}{2}) =$ 0.79, $f(\frac{5\pi}{6}) =$ 0.44

Find $f(\pi) =$ 0.57

(f) The graph of f is

QUESTION 3. Find all local maximums and minimums (if any) and all inflection points of the function $f(x) = xe^{2011x}$.

Note that $f'(x) = e^{2011x} + xe^{2011x}2011 = e^{2011x}(1 + 2011x)$. Note that $f'(x) = 0$ if and only if $x = \frac{-1}{2011}$. Since $f'(x) < 0$ for $x < \frac{-1}{2011}$ and $f'(x) > 0$ for $x > \frac{-1}{2011}$, it follows that $x = \frac{-1}{2011}$ is a local minimum.

Using again the product rule one may obtain that $f''(x) = 2011e^{2011x} + (1+2011x)e^{2011x}2011 = e^{2011x}2011(2 + 2011x)$, hence $f''(x) = 0$ if and only if $x = \frac{-2}{2011}$. Since $f''(x) < 0$ for $x < \frac{-2}{2011}$ and $f''(x) > 0$ for $x > \frac{-2}{2011}$, it follows that $x = \frac{-2}{2011}$ is an inflection point.

Space for the last question.