Final Exam CVG2116 - WINTER 2008

Hydrostatic Equation: $\int dp = \int \gamma dz \Leftrightarrow \Delta p = -\gamma \Delta z$

Piezometric pressure: $p + \gamma z = const. = C$;

Piezometric head: $\left(\frac{p}{\gamma} + z \right) = const. = C$

Hydrostatic Force

$$F = \int_A p dA = \overline{p} A$$

for a general case ($\overline{p}$ is the pressure at the centroid of the surface, A is the surface area)

$$y_{cp} = \overline{y} + \frac{\overline{I}}{\overline{y}A}; \quad x_{cp} = \overline{x} + \frac{\overline{I}_{xy}}{\overline{y}A}$$ (coordinates of the centre of pressure)

where $\overline{I}$ is the first moment of area with respect to the horizontal centroidal axis

$\overline{I}_{xy}$ is the product of inertia with respect to the centroidal axes

$\overline{y}$ and $\overline{x}$ are the coordinates of the surface centroid.

Surface Tension: $F_\sigma = \sigma L$

Elasticity: $E_v = \rho \frac{dp}{d\rho} = -\frac{dp}{dV/V}$

Shear Stress: $\tau = \mu \frac{du}{dy}$

Ideal Gas: $\rho = \frac{p}{RT}$

Buoyancy: $F_B = \gamma V_D \qquad GM = \frac{I_{00}}{V} - CG$

where I_{00} is the second moment, or moment of inertia of the area defined by the waterline

Euler's Equation: $-\frac{\partial}{\partial l}\left(p + \gamma z \right) = \rho a_l$ (where a_l is the fluid acceleration in the l direction)

Pressure Variation Equation for Rotating flow

$$p + \gamma z - \rho \frac{\omega^2 r^2}{2} = const. = C \qquad p + \gamma z - \rho \frac{V^2}{2} = const. = C$$

Bernoulli's Equation: $p + \gamma z + \rho \frac{V^2}{2} = const. = C$ ($\frac{p}{\gamma} + z + \frac{V^2}{2g} = C$)

Discharge: $Q = AV$; Mass flow rate: $\dot{m} = \rho Q$

Continuity Equation: $\frac{d}{dt}\int_{cv}\rho d\forall + \int_{cs}\rho \mathbf{V}\cdot d\mathbf{A} = 0$ or $\frac{dM_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out}$

for flow in pipes: $A_2V_2 = A_1V_1$, or $Q_2 = Q_1 \Rightarrow \sum_{cs} Q_{out} = \sum_{cs} Q_{in}$ or $\sum_{cs} \dot{m}_{in} = \sum_{cs} \dot{m}_{out}$

Momentum Equation: $\sum \mathbf{F} = \frac{d}{dt}\int_{cv}\mathbf{v}\rho d\forall + \int_{cs}\mathbf{v}\rho\mathbf{V}\cdot d\mathbf{A}$

$$\sum \mathbf{F} = \frac{d}{dt}\int_{cv}\mathbf{v}\rho d\forall + \sum_{cs}\dot{m}_o\mathbf{v}_o - \sum_{cs}\dot{m}_i\mathbf{v}_i$$

$$\sum \mathbf{F}_x = \frac{d}{dt}\int_{cv}\mathbf{v}_x\rho d\forall + \sum_{cs}\dot{m}_o\mathbf{v}_{ox} - \sum_{cs}\dot{m}_i\mathbf{v}_{ix} \qquad x \text{ direction}$$

$$\sum \mathbf{F}_y = \frac{d}{dt}\int_{cv}\mathbf{v}_y\rho d\forall + \sum_{cs}\dot{m}_o\mathbf{v}_{oy} - \sum_{cs}\dot{m}_i\mathbf{v}_{iy} \qquad y \text{ direction}$$

$$\sum \mathbf{F}_z = \frac{d}{dt}\int_{cv}\mathbf{v}_z\rho d\forall + \sum_{cs}\dot{m}_o\mathbf{v}_{oz} - \sum_{cs}\dot{m}_i\mathbf{v}_{iz} \qquad z \text{ direction}$$

Energy Equation (for steady flow): $\dot{Q} - \dot{W} = \sum_{CS}\dot{m}_o\left(\frac{V^2}{2g} + gz + h\right)_o - \sum_{CS}\dot{m}_i\left(\frac{V^2}{2g} + gz + h\right)_i$

Energy equation for steady flow of an incompressible fluid in a pipe:

$$\frac{p_1}{\gamma} + z_1 + \alpha_1\frac{V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \alpha_2\frac{V_2^2}{2g} + h_t + h_L$$

Power equation:

$\dot{W}_p = \gamma Q h_p = \dot{m} g h_p$ (pump)

$\dot{W}_t = \gamma Q ht = \dot{m} g h_t$ (turbine)

$\eta = \frac{P_{output}}{P_{input}}$ (efficiency)

Flow in conduits:

Laminar flow:

shear stress profile $\tau = \frac{r}{2}\left[-\frac{d}{ds}(p + \gamma z)\right]$

average velocity $V = \frac{r_o^2 - r^2}{4\mu}\left[-\frac{d}{ds}(p + \gamma z)\right]$

head losses due to frictional resistance of a pipe $h_f = \frac{32\mu LV}{gD^2}$

Turbulent flow $\rightarrow$ **Smooth pipes**

- velocity profile $\frac{u}{u_*} = 5.75\frac{1}{\kappa}\log\frac{yu_*}{\nu} + 5.5$

- bed shear stress $\tau_o = \frac{f}{4}\rho\frac{\bar{V}^2}{2}$
- shear velocity $u_* = \sqrt{\tau_o / \rho}$
- friction coefficient $\frac{1}{\sqrt{f}} = 2\,\mathbf{log}\left(\mathbf{Re}\sqrt{f}\right) - 0.8$ for $\mathbf{Re} > 3000$

→ **Rough pipes**

- velocity profile $\frac{u}{u_*} = 5.75\frac{1}{\kappa}\mathbf{log}\frac{y}{k_s} + 8.5$
- friction coefficient $f = \frac{0.25}{\left[\mathbf{log}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathbf{Re}^{0.9}}\right)\right]^2}$

$$\mathbf{Re}\, f^{1/2} = \frac{D^{3/2}}{n}\left(\frac{2gh_f}{L}\right)^{1/2}$$

- discharge $Q = -2.22D^{5/2}\sqrt{gh_f / L}\,\mathbf{log}\left(\frac{k_s}{3.7D} + \frac{1.78\nu}{D^{3/2}\sqrt{gh_f / L}}\right)$
- pipe diameter $D = 0.66\left[k_s^{1.25}\left(\frac{LQ^2}{gh_f}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_f}\right)^{5.2}\right]^{0.04}$

Head losses due to frictional resistance of a pipe: $h_f = f\frac{L}{D}\frac{V^2}{2g}$

Head losses due to local disturbances in a pipe: $h_L = K_L\frac{V^2}{2g}$, where K_L is the local loss coefficient

Head-loss due to sudden-expansion: $h_L = \frac{(V_1 - V_2)^2}{2g}$

If the pipe discharges into a reservoir, then $V_2 = 0$ and the above expression simplifies to: $h_L = \frac{V^2}{2g}$

Properties of water at 20°C

μ = 1 x 10^{-3} N.s/m^2; υ = 1 x 10^{-6} m^2/s; E_v = 2.2 GN/m^2;

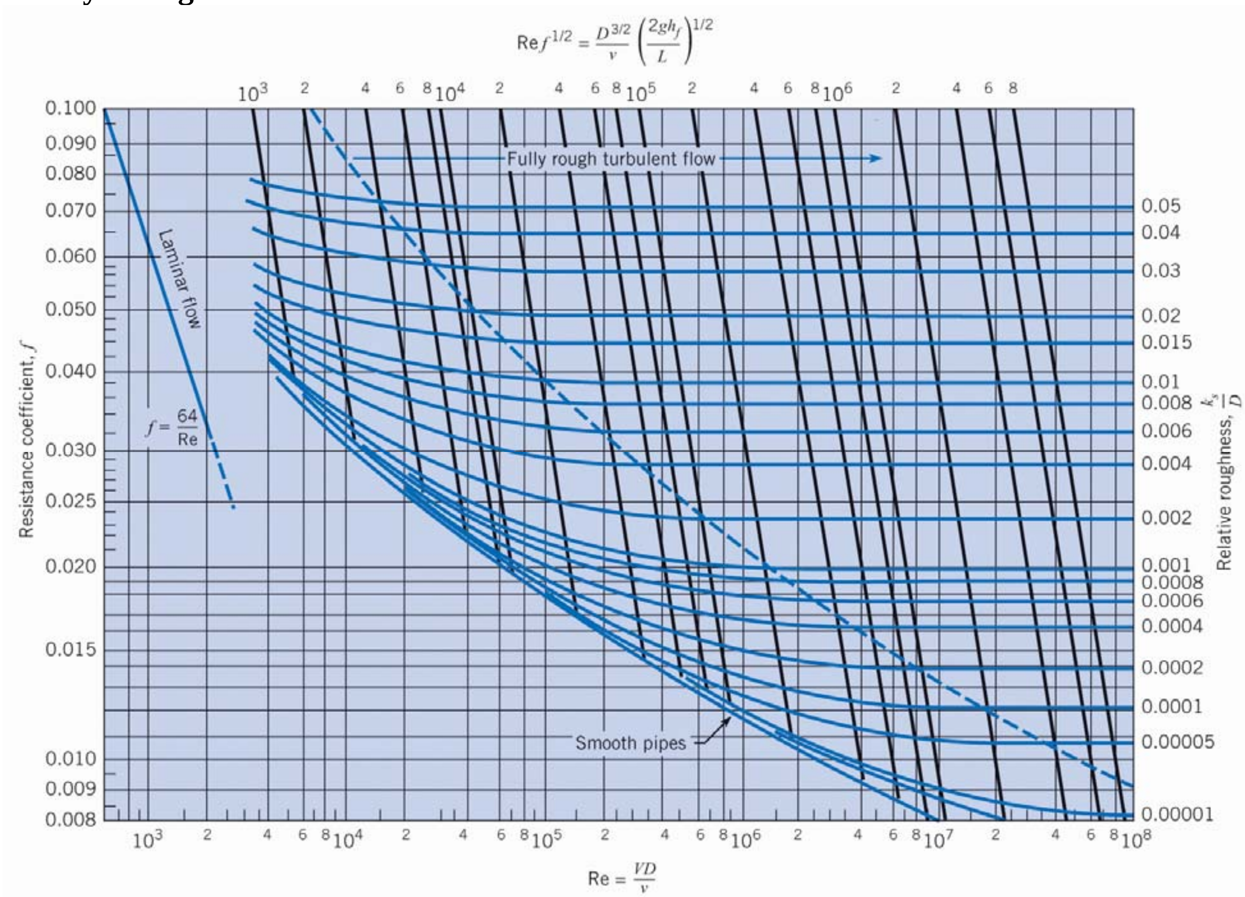
$\sigma_{water / Air}$ = 0.073 N/m; ρ = 998 kg/m^3 [1.94 slugs/ft^3]; γ = 9790 N/m^3;

p_{vapour} = 2,340 N/m^2 (abs.)

Conversions

1 ft = 0.3048 m; 1 atm = 101.3 kPa; 1 kg = 2.205 lbm 1 slug = 14.59 kg

Moody's Diagram



Moments of inertia and areas for various geometrical shapes

Figure A.1

Centroids and moments of inertia of plane areas.