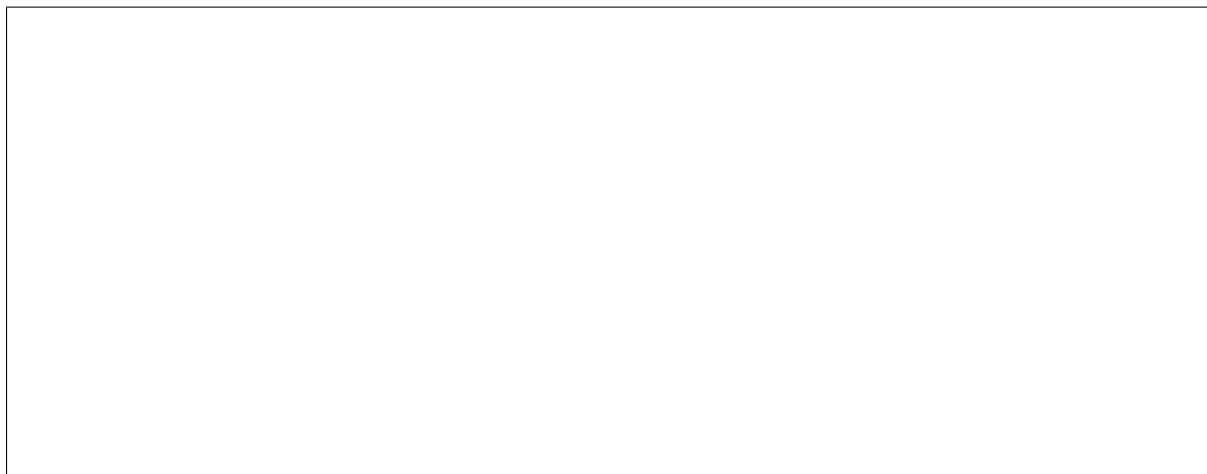


QUESTION 2. Find the global minimum and the global maximum of $f(x) = x - \ln x$ on the interval $[0.1, 2]$.



Global maximum at $x =$.

Global minimum at $x =$.

QUESTION 3.

Consider the following DTDS:

$$x_{t+1} = \frac{1.5x_t^2}{x_t^2 + 0.5}.$$

- (a) The DTDS has three equilibrium points. Give the equilibrium points in increasing order.

$x_1^* =$

$x_2^* =$

$x_3^* =$

- (b) Find the derivative of the updating function of the DTDS.

$f'(x) =$

(c) For each of the equilibrium points in (a) find $f'(x_i^*)$ and decide whether the equilibrium point is stable or unstable (circle the correct answer).

$$f'(x_1^*) = \boxed{} \quad \text{stable} \quad \text{unstable}$$

$$f'(x_2^*) = \boxed{} \quad \text{stable} \quad \text{unstable}$$

$$f'(x_3^*) = \boxed{} \quad \text{stable} \quad \text{unstable}$$

QUESTION 4.

The number of individuals (in thousands) of a certain species satisfies the DTDS:

$$x_{t+1} = \frac{4x_t}{1+x_t} - hx_t, \quad t = 0, 1, 2, \dots$$

The population is harvested with the rate $h \geq 0$. Answer the following questions:

(a) The equilibrium points of this DTDS are (Hint: one of the equilibrium points will depend on h):

and

(b) Give the largest interval for h such that both equilibrium points in (a) are non-negative (i.e. biologically meaningful).

$h \in$

(c) What is the harvest at the positive equilibrium? (Hint: the answer will depend on h)

$R(h) =$

(d) The maximal harvest at the positive equilibrium is

$R_{max} =$ and occurs for the harvesting rate $h =$

(e) For the values of h as in (b) let x^* be the positive equilibrium point. Then:

x^* is stable when h is

x^* is unstable when h is