MAT 1339 Summer 2012 July 30 , 14:30 - 16:00 Instructor I. Abdelrazeq

Midterm II

Max = 30+6

Name: Solution

Student Number: ______________________

- Time: 80 min.
- Only basic scientific calculators are permitted: non-programmable, non-graphing, no differentiation or integration capability. Notes or books are not permitted.
- Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other papers.
- Write only in non-erasable ink (ball-point or pen), not in pencil. Cross out, if necessary, but do not erase or overwrite. Graphs and sketches may be drawn in pencil.
- Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.

Problem 1: (8 marks) Banana computer sells 5000 laptops per month at \$1000. When the laptop price is raised to \$1100, the sales decline to 4000 per month

(a) (2 marks) We assume that demand function is linear, Determine the demand (price) function?

(b) (2 marks) What price should the company charge to maximize revenue?

(c) (2 marks) Suppose that the cost function is given by $C(x) = -0.02x^2 + 200x + 5000$. Determine the marginal cost at x=1000?

(d) (2 marks) Determine the marginal profit at x = 1000 ?

(a) We have two points $(5000, 1000)$, $(4000, 1100)$

$$\Rightarrow p - 1000 = \frac{-100}{1000}(x - 5000)$$

$$\Rightarrow p = \boxed{\frac{-1}{10}x + 1500}$$

(b) $R = xp(x) = \frac{-1}{10}x^2 + 1500x$

$R' = \frac{-1}{5}x + 1500 = 0 \Rightarrow \frac{1}{5}x = 1500$

$x = 7500$

$p(7500) = \frac{-1}{10}(7500) + 1500$

$= -750 + 1500 = \boxed{750}$ \$

(c) $c'(x) = -0.04x + 200$

$c'(1000) = -40 + 200 = \boxed{160}$

(d) $P(x) = R(x) - C(x)$

$P'(1000) = R'(1000) - C'(1000)$

$= -0.2(1000) + 1500 - 160$

$= 1300 - 160 = \boxed{1140}$

Problem 2: (10 marks) Let $f(x) = \frac{-1}{x(x-2)}$.

(a) (2 marks) Determine the domain of the function.

(b) (2 marks) Find the vertical and horizontal asymptotes.

(c) (2 marks) Calculate $f'(x)$ and find the critical points.

(d) (2 marks) Calculate $f''(x)$ and determine the intervals where $f(x)$ is concave up, concave down.

(e) (2 marks) Sketch the graph of the function.

(a) $\boxed{\mathbb{R} \setminus \{0, 2\}}$

(b) $x(x-2) = 0 \Rightarrow x = 0, \; x = 2$

$\boxed{\{x = 0, \; x = 2\}}$ vertical asymptotes

$\lim_{x \to \pm\infty} f(x) = 0 \Rightarrow \boxed{y = 0}$ is a horizontal Asymptotes

(c) $f'(x) = \dfrac{2x-2}{(x^2-2x)^2} \Rightarrow x = \{1\}$ is a critical point

$2x - 2 = 0 \Rightarrow \boxed{x = 1}$

Also we consider $x = 0$, & $x = 2$

They make $f'(x)$ does not exist.

(d) $f''(x) = \dfrac{2(x^3-2x)^2 \; (2x-2) \cdot 2(x^2-2x)(2x-2)}{(x^2-2x)^4}$

$= \dfrac{2(x^2-2x)(-3x^2+6x-4)}{(x^2-2x)^4}$

Now, $-3x^2+6x-4 = 0$ has no roots $(\nabla < 0)$,

$f''(0) \neq 0$ but we consider $x = 2, \; x = 0$

	$(-\infty, 0)$	$(0, 1)$	$(1, 2)$	$(2, \infty)$
f'	$-$	$-$	$+$	$+$
f''	$-$	$+$	$+$	$-$
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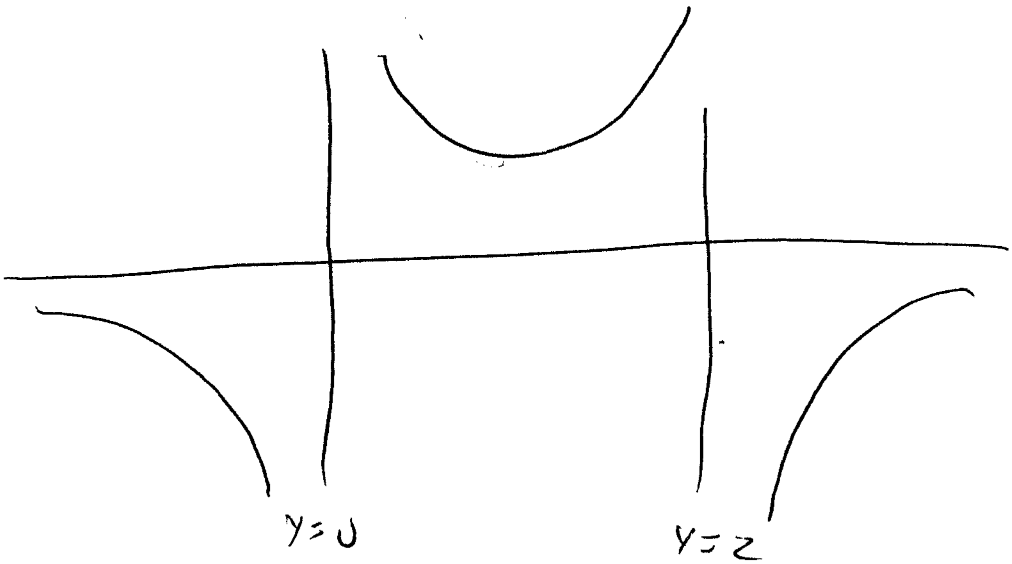
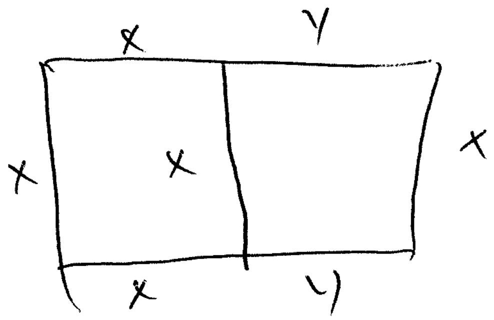


Problem 3: (6 marks) Two pens with one common side are to be built with 60 m of fencing. One pen is to be square, the other rectangular. Find the dimensions that maximize the total area.



$$60 = 5x + 2y$$

$$\frac{60-5x}{2} = y$$

2/6

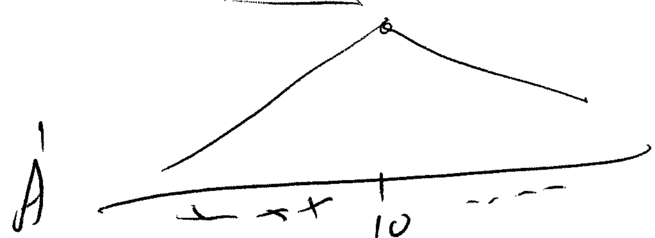
$$A = x^2 + xy$$

$$= x^2 + x\left[\frac{60-5x}{2}\right]$$

$$= x^2 + 30x - \frac{5}{2}x^2$$

$$= -\frac{3}{2}x^2 + 30x$$

$$A' = -3x + 30 \Rightarrow \boxed{x = 10}$$



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$$x = 10, \Rightarrow y = \frac{60-50}{2} = 5$$

2/6

Dimension is 10 x 15 m.

Problem 3: (6 marks) Find the following derivatives:

(a) (2 marks) Let $f(x) = 4x\cos(2x)$, find $f'(\frac{\pi}{6})$.

(b) (2 marks) Let $f(x) = e^{\tan x}$, find $f'(\frac{\pi}{3})$.

(c) (2 marks) Let $f(x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x}$, find $f'(\frac{\pi}{4})$.

(a) $f'(x) = -8x \sin(2x) + 4\cos(2x)$

$f'(\frac{\pi}{6}) = -8\frac{\pi}{6}\frac{\sqrt{3}}{2} + \frac{4}{2}$

$= \boxed{2 - \frac{2}{\sqrt{3}}\pi}$

(b) $f'(x) = \left(\frac{\sin x}{\cos x}\right)' e^{\tan x} = \frac{\cos x \cos x + \sin x \sin x}{\cos^2 x} e^{\tan x}$

$= e^{\tan x} \frac{1}{\cos^2 x}$

$f'(\frac{\pi}{3}) = e^{\sqrt{3}} \frac{1}{(\frac{1}{2})^2} = \boxed{4e^{\sqrt{3}}} \approx 22.61$

(c) $f(x) = \frac{1}{\cos 2x}$, $f'(x) = \frac{2\sin 2x}{\cos^2 2x}$

$f'(\frac{\pi}{4}) = \boxed{\frac{2}{0}}$ undefined or $\boxed{\text{D.N.E.}}$

Problem 4: (6 marks) Suppose that two functions $f(x)$ and $g(x)$ are inverse functions of one another such that :

(a) (2 marks) $f(x) = \ln(x^3 + 7)$, find $g'(1)$.

(b) (2 marks) $f'(2) = 5$, $f(2) = 7$, find $g'(7)$.

(c) (2 marks) $f(x) = x + e^x$ find, $g'(1)$.

(a) $y = \ln(x^3+7) \Leftrightarrow x = \ln(y^3+7) \Rightarrow e^x = y^3+7$

$y = \sqrt[3]{e^x - 7} \Rightarrow g(x) = \sqrt[3]{e^x - 7}$

$g'(x) = \frac{1}{3}(e^x - 7)^{-\frac{2}{3}} e^x$

$g'(1) = \boxed{\frac{1}{3} \frac{1}{\sqrt[3]{(e-7)^2}}} = 0.3437$

(b) $g'(7) = \frac{1}{f'(g(7))} = \frac{1}{f'(2)} = \boxed{\frac{1}{5}}$

(c) $g'(1) = \frac{1}{f'(g(1))}$, $y = x + e^x$

$\Leftrightarrow x = y + e^y$

when $x = 0 \Rightarrow 1 = y + e^y$

$\Rightarrow y\big|_{x=1} = 0$

$y = 0$

$g(1) = 0$

$g'(1) = \frac{1}{f'(g(1))}$

$= \frac{1}{f'(0)} = \frac{1}{2}$

End of problems
Good luck
I.A.