MAT 1330, Fall 2011 Assignment 1

Due September 28 at the beginning of class.

Late assignments will **not** be accepted; **nor** will unstapled assignments.

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Student Name _____ Student Number _____

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature _____

QUESTION 1.

A patient's metabolism will break down morphine to 60% of its original amount every day. The patient is given 8 units of morphine to raise to concentration of the medication at the end each day. Let M_t denote the concentration of morphine at time t.

(a) Write down the associated DTDS and indentify the updating function.

(b) If the patient body contains $M_t = 5$ units of morphine at the start of the day, write down the amount of the medication at the end of the day and at the end of the next day.

End of the day: End of the next day:

(c) Compose the updating function with itself and find the two step DTDS.

(d) Find the backwards DTDS (the inverse of the updating function) and use it to find the value at the previous time ($t = 5$), knowing that $M_6 = 11$.

Value at $t = 5$:

(e) Find the equilibrium point of the dynamical system.

Equilibrium point:

(f) Find the general solution algebraically and use the formula to find the concentration of medication at $t = 1, 2, 3, 4$ starting at $M_0 = 0$.

t	1	2	3	4
M_t				

General solution:

(g) Draw the solution of the DTDS with the four iterations obtained in (f).

(i) Draw the cobweb diagram of the DTDS starting with $M_0 = 0$ (four iterations are enough).

(j) Determine the stability of the equilibrium point using the cobweb diagram.

Answer:

QUESTION 2. For a bacteria colony that takes 3 hours to double in size, it took the colony 12 hours to reach the size of 2,000,000 bacteria. How long did it take to reach 1,000,000?

Answer:

QUESTION 3. Consider an experiment where salt crystals are grown in a super-saturated solution. The mass of a crystal grows over 24 hours based on the updating function

$$m_{t+1} = f(m_t) = 1.5m_t.$$

Given that the crystal originally has a mass of 10 grams, express the solution to this system, both in a table (four calculations are enough), graphically and as a formula.

QUESTION 4. A population grows according to the formula $b_{n+1} = \dfrac{3b_n}{1 + 0.2b_n}$.

(a) Find all the equilibrium points of the DTDS.

Equilibrium points:

(b) Assume that we start with $b_0 = 100$. What will be the size of the population after the 1st, 2nd, and 3rd year.

(c) Assume that we start with $b_0 = 10$. What will be the size of the population b_1, b_2, b_3?

(d) Assume that we start with $b_0 = 3$. What will be the size of the population b_1, b_2, b_3?

(e) Sketch the solution curves obtained in $(b), (c), (d)$ in the same tb_t-plane. Give short comments about the above results (three sentences are enough).