Kruskal-Wallis Test

Used to test if there is a difference between MEDIANS for treatment groups
Use for NON-NORMAL distributions
Assumptions:

1) Completely Randomized Design (CRD)
2) Treatment populations have approximately the same shape and spread

Based on RANKING ALL SAMPLES from smallest to largest. Then take MEDIANS (Md)

H_o: $Md_1 = Md_2 = Md_3 \ldots = Md_k$;
H_a: At least ONE of the Md's is ≠

test statistic: $\mathbf{H} = \frac{12}{N(N+1)} * \left[\sum_{i=1}^{k} \frac{(Tr_i)^2}{n_i}\right] - (3(N+1))$

$\boldsymbol{k}$ = # of treatment groups
$\boldsymbol{N} = \sum_{i=1}^{k} n_i$ (Total number of all samples)

Rejection Region: Reject H_o if $\mathbf{H} > \boldsymbol{X^2}_{\boldsymbol{\alpha;(k-1)}}$ **(chi-squared distribution)**

Steps:

1) **Rank all observations from smallest to largest (easiest if you can order them within a treatment group)**
2) **If two or more observations are "tied" create and assign a fractional rank for them, i.e.**

Scores =	**2**	**5**	**6**	**6**	**6**	**10**	**12**	
Initial Rank =	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**(3+4+5)/3 = 4**
Final Rank =	**1**	**2**	**4**	**4**	**4**	**6**	**7**	

3) **Sum all the ranks: i.e.**

Tr_A	**66**	**73**	**74**	**75**	**78**	**82**	**87**	**97**	
RANK	**15**	**20**	**21.5**	**23**	**25.5**	**29**	**31**	**32**	**Tr_A = 197**
Tr_B	**51**	**59**	**62**	**63**	**64**	**72**	**74**	**78**	
RANK	**1**	**5**	**10**	**11.5**	**13**	**19**	**21.5**	**25.5**	**Tr_B = 106.5**
Tr_C	**55**	**57**	**60**	**60**	**61**	**70**	**71**	**81**	
RANK	**2**	**3**	**6.5**	**6.5**	**8.5**	**17**	**18**	**28**	**Tr_C = 89.5**
Tr_D	**58**	**61**	**63**	**65**	**69**	**76**	**80**	**84**	
RANK	**4**	**8.5**	**11.5**	**14**	**16**	**24**	**27**	**30**	**Tr_D = 135**
							Total		**528**

4) **Check THE RANKING total:** $\sum_{i=1}^{k} Tr_i \ \ should = \frac{N*(N+1)}{2}$ **= 32(33)/2 = 528**

5) **Calculate the test statistic:**

$$H = \frac{12}{N(N+1)} * \left[\frac{197^2}{8} + \frac{106.5^2}{8} + \frac{89.5^2}{8} + \frac{135^2}{8}\right] - (3(32+1)) = 9.50355136$$

6) **Compare to Chi-squared value:**

$X^2{}_{\alpha;(k-1)}$ = $X^2{}_{0.10;(3)} = 6.25139$

7) Write Conclusion:
Since H = 9.50355136 > $X^2{}_{\alpha;(k-1)}$ = 6.25139 we can reject H_o and conclude that at a 10% level of significance there are differences between the treatments.

Dunn's procedure – Test which treatment groups differ, based on MEDIANS

Assumptions:

Calculate $\binom{k}{2}$ pairs of $\left|\bar{R}_i - \bar{R}_j\right|$

H_o: $Md_i = Md_j$; where i,j = 1:k and i≠j
H_a: $Md_i \neq Md_j$; where i,j = 1:k and i≠j

$\boldsymbol{k} = \#\ of\ treatment\ groups$; $\boldsymbol{R}_i = \frac{\boldsymbol{Tr_i}}{\boldsymbol{n_i}}$;
$\boldsymbol{N} = \sum_{i=1}^{k} n_i$ (Total number of all samples)

Critical region, $CR = Z_{\frac{\alpha}{k(k-1)}}\left(\sqrt{\frac{N(N+1)}{12} * \left(\frac{1}{n_i} + \frac{1}{n_j}\right)}\right)$

Reject H_o ($Md_i = Md_j$) if $\left|\bar{R}_i - \bar{R}_j\right| > CR$

eg. if α = 0.1 and k = 4, then $\frac{\alpha}{k(k-1)} = \frac{0.1}{4(3)} = 0.00833333$

Look in NORMAL tables (Area Under The Normal Curve) in textbook:
1 - 0.0083333 = 0.99166667 ← look up this number

In tables, Z(2.39) = 0.9916 and Z(2.4) = .9918 so use Z = 2.395

Steps:

1) Calculate $\bar{R}_i$ for each Treatment: $\bar{R}_i = \frac{Tr_i}{n_i}$ i.e.:

$$\overline{R_1} = \frac{63.5}{6} = 10.583$$
$$\overline{R_2} = \frac{89}{7} = 12.7142$$
$$\overline{R_3} = \frac{45.5}{6} = 7.5833$$
$$\overline{R_4} = \frac{78}{4} = 19.5$$

2) Calculate pairs of $\left|\overline{R_i} - \overline{R_j}\right|$ eg:

$|\overline{R_1} - \overline{R_2}| = 2.13$ $\ngtr$ 9.037
$|\overline{R_1} - \overline{R_3}| = 2.999$ $\ngtr$ 9.3782
$|\overline{R_1} - \overline{R_4}| = 8.9167$ $\ngtr$ 10.4852
$|\overline{R_2} - \overline{R_3}| = 5.13095$ $\ngtr$ 9.037
$|\overline{R_2} - \overline{R_4}| = 6.7857$ $\ngtr$ 10.81265
$|\overline{R_3} - \overline{R_4}| = 11.9166$ $>$ 10.4851

3) Calculate Critical region for EVERY PAIR, ***unless n_i = nj for all***

$$CR = Z_{\frac{\alpha}{k(k-1)}}\left(\sqrt{\frac{N(N+1)}{12} * \left(\frac{1}{n_i} + \frac{1}{n_j}\right)}\right)$$

4) Compare EACH pair value against the CR (**see #2 above**)
5) REJECT H_o if Pair value > CR
6) WRITE CONCLUSION:
There is evidence to suggest that at a 10% level of significance there are differences between Treatments 3 and 4.

(There is NO SAS program equivalent for this – so no need to compare to SAS output)

NOTES:
Non-Parametric tests are more powerful (greater probability of rejecting a false Ho) than the F-Test, IF the assumptions of the F-Test are violated (i.e. NON-NORMALITY). HOWEVER, if F-Test assumptions are REASONABLY SATISFIED, the F-Test is better than non-parametric tests at detecting small but significant differences between treatment means.