NAME:

STUDENT NUMBER:

ECO 3153
Winter 2013

Final Exam
April 23rd, 2013

Instructions:

1. All questions should be answered on the questionnaire. Use the back of the pages as scrap paper.

2. Only nonprogrammable calculators are permitted during this exam.

3. The marks for each question are given in bold following the question. Budget your time accordingly.

4. The maximum grade is **100**.

5. This exam consists of **14** pages and **4** questions. It is your responsibility to ensure that your exam questionnaire is complete.

5. The exam ends at 12:30.

6. Good luck!

Question 1	/24
Question 2	/36
Question 3	/19
Question 4	/21
Total	/100

Question 1

You are stuck alone on an island. Only two goods are available xeruses (x) – some sort of ground squirrels – and yams (y) for consumption. Your production possibility frontier is represented by

$$x^2 + 5y^2 \leq Q$$

where $Q > 0$. Your utility function is given by

$$U(x, y) = \alpha \ln x + (1 - \alpha) \ln y$$

where $0 < \alpha < 1$.

a) Set up the Lagrangian for the utility maximizing problem. **2 points**.

b) Show that the optimal consumption of yams is $y^* = \sqrt{\frac{Q(1-\alpha)}{5}}$. **5 points**.

c) If you were to invent prices for xeruses and yams that would be consistent with the solution above, what would be the price ratio ρ_x/ρ_y? **5 points**.

d) You can now trade with the rest of the world. The international prices of xeruses and yams are 1 and 2.5 dollars, respectively. Show that the optimal *production* of xeruses and yams will be $2\sqrt{Q}/3$ and $\sqrt{Q}/3$, respectively. **5 points**.

e) At the international prices, how much xeruses and yams would you import/export? Hint: The answers are functions of α and Q. **7 points**.

Question 2

Assume you want to maximize the profit of your (single-product) firm. We have seen in class that the firm profit maximization problem can be broken into two stages: 1) cost minimization and 2) output choice.

a) Write down the output choice problem using the notation seen in class. **2 points**.

b) Show that, if we have an interior solution, the marginal benefit of producing q will be equal to its marginal cost. **6 points**.

c) Show that the second-order condition for profit maximization implies that the marginal-cost curve is upward sloping at q^*. **6 points**.

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d) Assume that the cost function is convex in q. Show that the output supply curve (obtained from the second stage of the profit maximization problem), $S(\mathbf{w}, p)$, is upward sloping in p. **6 points.**

e) Assume an interior solution for the output-choice problem. Show what will be the impact of an increase in the price of input j (i.e. w_j) on the output supply (i.e. $S(\mathbf{w}, p)$), and show under which condition the output supply will fall as w_j increases. **6 points.**

f) Show that the effect of an increase of w_i on $D^i(\mathbf{w}, p)$ can be separated into a substitution and an output effects, and that $D^i(\mathbf{w}, p)$ cannot rise as w_i increases. **10 points.**

Question 3

Let a consumer's utility function be

$$U(\mathbf{x}) = \prod_{i=1}^{n} x_i^{\beta_i}$$

where $0 < \beta_i < 1$ and $\sum_{i=1}^{n} \beta_i = 1$. Assume that the consumer's fixed income is y.

a) Write down the Lagrange function associated to this utility-maximization problem. **2 points**.

b) Find the marginal utility of income for this problem. **2 points**.

c) Show that the Marshallian demand function for good i is

$$D^i(\mathbf{p}, y) = \frac{y\beta_i}{p_i}$$

7 points.

d) *Without solving the cost-minimization problem*, find the Hicksian demand function for good i? **8 points**.

Question 4

You, a risk averse agent with wealth $\bar{y}$, decide to rent a car to drive to Toronto. The cost of renting the car is ρ. There is a possibility π that you will damage the car. The cost of repairing the car is $R < \bar{y} - \rho$. The car rental company offers coverage (an insurance) for the repair cost at a premium $\phi > \pi R$. It is possible to take out partial cover on a pro-rata basis, so that an amount tR of the repair cost can be covered at a cost $t\phi$ where $0 \leq t \leq 1$. Assume that your utility can be written in a vNM form.

a) What is your expected utility if you decide not to take the insurance? **2 points**.

b) Assuming an interior solution, show that the condition that will determine the optimal value of t, t^* is given by

$$\phi(1-\pi)u_y(\bar{y} - \rho - t^*\phi) = (\phi - R)\pi u_y(\bar{y} - \rho - t^*\phi - (1-t^*)R)$$

where y is your ex-post wealth. **4 points**.

c) Explain why you will not choose full insurance. **5 points**.

d) Show how t^* will change as $\bar{y}$ increases. **10 points**.