NAME:

STUDENT NUMBER:

**ECO 3153**
**Winter 2013**

**Midterm 2**
**March 14$^{th}$, 2013**

Instructions:

1. All questions should be answered on the questionnaire. Use the back of the pages as scrap paper.

2. Only nonprogrammable calculators are permitted during this exam.

3. The marks for each question are given in bold following the question. Budget your time accordingly.

4. The maximum grade is **100**.

5. This exam consists of **12** pages and **5** questions. It is your responsibility to ensure that your exam questionnaire is complete.

6. Good luck!

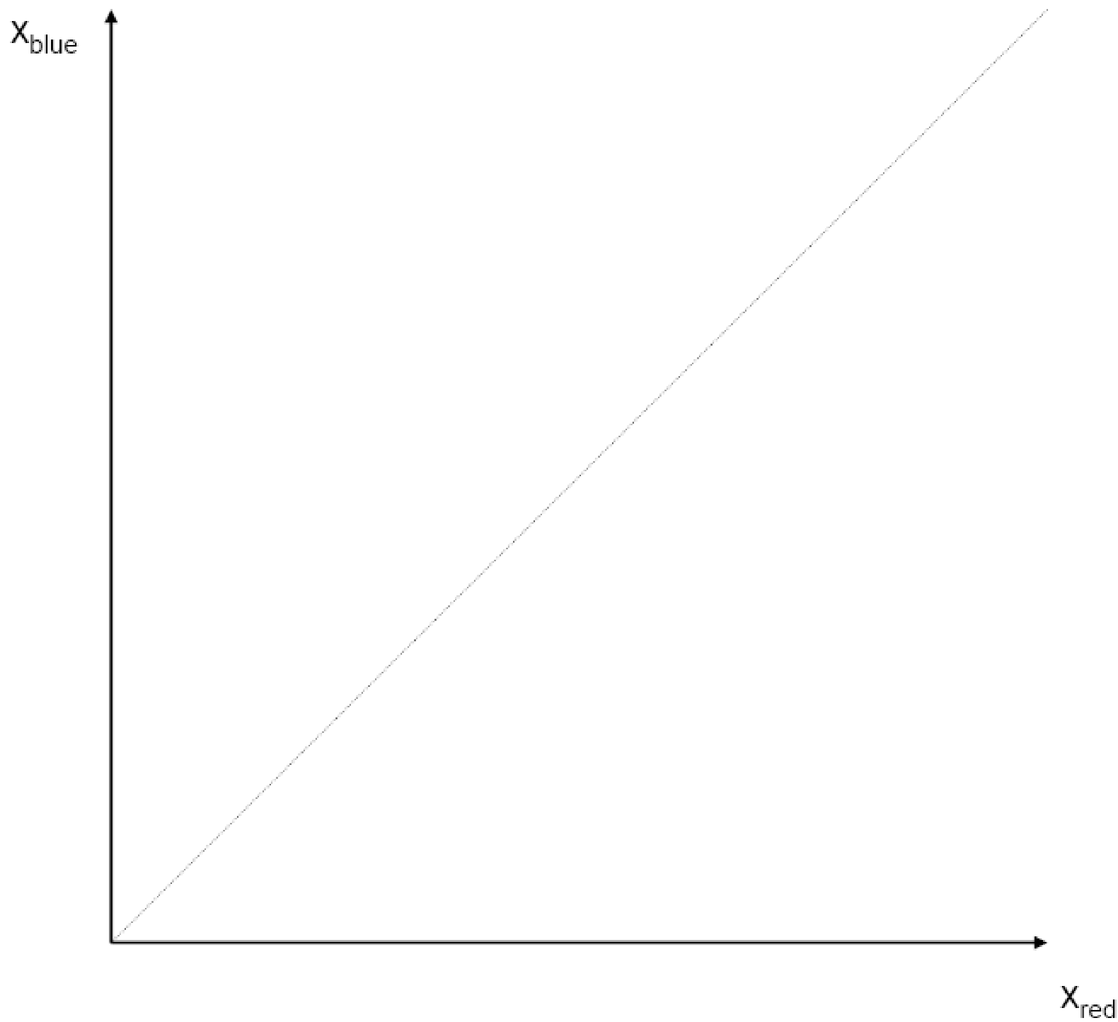
| | |
|---|---|
| Question 1 | /30 |
| Question 2 | /15 |
| Question 3 | /20 |
| Question 4 | /15 |
| Question 5 | /20 |
| Total | /100 |

## Question 1

a) List the three key axioms about one's preferences necessary, in the presence of uncertainty, to represent these preferences in an expected utility form (these assumptions are not necessary in the absence of uncertainty). **6 points**.

1. ________________
2. ________________
3. ________________

b) Assume that an individual's preferences conform to the vNM configuration, that there are only two possible states of the world (blue or red), and that the payoff in each state is a scalar. On the graph on page 3, draw the indifference curves of an individual who is risk averse, and illustrate what would happen to the indifference curves if, suddenly, the subjective probability she attributes to "state red occurs" increases. **6 points**.


$x_{blue}$
$x_{red}$

c) Show that the slope of the indifference curves on the graph on page 3 is equal to $-\pi_{red}/\pi_{blue}$ on the 45 degree line, where $\pi_{red}$ and $\pi_{blue}$ represent the subjective probabilities of state red and state blue, respectively. **9 points**.

d) Using a graph similar to the one on page 3 (and some algebra), show that the felicity function of a risk averse individual must be concave. **9 points**.

## Question 2

Suppose you take pleasure in being famous. The utility you get from your fame level, $x$, can be described by the following felicity function

$$u(x) = \sqrt{x}$$

You contemplate the possibility of entering the 'So You Think You Can Dance Canada' contest. Your current fame level is 144. You imagine three possible scenarios if you enter the competition. First, you think that there is an eighty (80) percent chance that the juries don't select you to go on tv. In this case, your fame level stays unchanged. Second there is a ten (10) percent chance that you are selected to appear on tv and win the contest. In this scenario, your fame level would jump to 361. Finally, there is ten (10) percent chance that you are selected to appear on tv and that you fall on your face while dancing on tv. In this case, your fame level would drop to 0.

a) What is the certainty equivalent of the 'So You Think You Can Dance Canada' contest? **5 points**.

b) What is the risk premium? **5 points**.

c) Should you enter the competition? Why? **5 points**.

## Question 3

a) Suppose you are asked to choose between two lotteries. In one case the choice is between $P_1$ and $P_2$, and in the other case the choice offered is between $P_3$ and $P_4$, as specified below:

$$
\begin{aligned}
P_1 &: \$1,000,000 \text{ with probability } 1 \\
P_2 &: \begin{cases} \$5,000,000 \text{ with probability } 0.1 \\ \$1,000,000 \text{ with probability } 0.89 \\ \$0 \text{ with probability } 0.01 \end{cases} \\
P_3 &: \begin{cases} \$5,000,000 \text{ with probability } 0.1 \\ \$0 \text{ with probability } 0.9 \end{cases} \\
P_4 &: \begin{cases} \$1,000,000 \text{ with probability } 0.11 \\ \$0 \text{ with probability } 0.89 \end{cases}
\end{aligned}
$$

You prefer $P_1$ to $P_2$ and then $P_3$ to $P_4$. Show that the independence axiom is violated here. **10 points**.

b) Imagine there are two urns marked L and R, each of which contain 100 balls. In Urn L there are 49 white balls and 51 black balls. and in Urn R there are black and white balls, but in unknown proportions. Consider the following experiments:

1. One ball is drawn from each of L and R. You choose between L and R before the draw is made. If the ball drawn from the chosen urn is black you win \$100, otherwise you win 0\$.
2. One ball is drawn from each of L and R. You choose between L and R before the draw is made. If the ball drawn from the chosen urn is white you win \$100, otherwise you win 0\$.

You choose Urn L in both experiments. Show that the revealed-likelihood axiom is violated here. **10 points**.

## Question 4

Suppose you come to my office at the end of the semester hoping to get a better grade. Your actual numerical mark is $W$ and assume you exhibit risk aversion with respect to this mark. I offer you two possible 50-50 gambles. Gamble no.1: a 50-50 chance of winning or losing $h$ marks. Gamble no.2: a 50-50 chance of winning or losing $2h$ marks.

a) Show that the two gambles have the same expected outcome. **5 points**.

b) Illustrate these two gambles on a graph containing your felicity function and show that you prefer gamble no.1 (since you are risk averse) by showing that the certainty equivalent of gamble no.1 is larger than the certainty equivalent of gamble no.2. **10 points**.

# Question 5

You are studying for your ECO3153 final exam. You know that your grade on the final exam, $g$, depends on your effort level, e, and on the professor generosity, $\varepsilon$. More specifically, the grade is determined by the following equation

$$g(e) = f(e) + \varepsilon$$

where $f(e)$ is a strictly increasing and strictly concave function of e (i.e. $f'(e) > 0$ and $f''(e) < 0$) and where $f(0) = 0$. The professor's generosity is a random variable such that $E(\varepsilon) = \mu > 0$ and $var(\varepsilon) = \sigma^2$.

Your felicity function has the following shape:

$$u(x) = \beta_0 + \beta_1 g(e) + \beta_2 [g(e)]^2 - C(e)$$

where $\beta_1 > 0$, $\beta_2 < 0$, and $C(e)$ is the cost of exerting effort. $C(e)$ is deterministic (i.e. not random) and increasing and convex in $e$.

a) Show that your expected utility can be written as

$$E(u(x)) = \beta_0 + \beta_1 \mu + \beta_2 [\sigma^2 + \mu^2] + [\beta_1 + 2\mu\beta_2] f(e) + \beta_2 [f(e)]^2 - C(e)$$

**5 points**.

b) Assuming an interior solution, show that the condition that will determine the optimal value of $e$, $(e^*)$ is given by

$$[\beta_1 + 2\mu\beta_2]f'(e^*) + 2\beta_2 f(e^*)f'(e^*) = C'(e^*)$$

**5 points**.

c) Show the conditions under which you will exert a strictly positive amount of effort (i.e. $e^* > 0$). **5 points**.

d) Write the expression that would determine the effect of increasing the professor's average generosity (say by adding $\tau$) on the optimal effort level. Assume an interior solution. **5 points**.