Question 1. [8 marks]

(a) [3] H_o: $\mu_B - \mu_A = \mu_d = 0$; H_a: $\mu_d > 0$; $t = 4.0/(6.0/\sqrt{9}) = 2.0$,

Reject H_o if $t > t(.05,8) = 1.860$ ➔ reject H_o. There is sufficient evidence to conclude that Food B gives greater weight gain than Food A.

1 mark for hypothesis, 1 mark for test statistic, ½ mark for critical value or p-value, ½ for conclusion

[if $\mu_A - \mu_B = \mu_d$, then H_a: $\mu_d < 0$ and t = -2. Then reject H_o if $t < -t(.05,8) = -1.860$]

(b) [4] H_o: $\mu_B - \mu_A = 0$; H_a: $\mu_B - \mu_A > 0$; $s_p = 8.7$; $t = (50 - 46)/\ 8.7\sqrt{(1/9+1/9)} = 4.0/4.1 = 0.98$

Reject H_o if $t > t(.05,16) = 1.746$ ➔ accept H_o. There is not enough evidence to conclude that Food B gives greater weight gain than Food A.

1 mark for hypotheses, 1 mark for test statistic, 1 mark for critical value or p-value, 1 mark for conclusion

(c) [1] In the second test the variation between dogs masks the variation between dog foods.

Question 2 (11 marks)

a. H0: The distribution of the types of TVs sold is the same in the various provinces
   Ha: The distribution of the types of TVs sold is not the same in the various provinces

The hypotheses below are acceptable, but keep in mind that we have four samples not one sample.

H0: The types of TVs sold and the provincial location are independent variables
Ha: The types of TVs sold and the provincial location are not independent variables [1]

$f_{.2} = 175$, $f_{2.} = 214$, n = 495, df = (r – 1)*(c-1) = (4-1)*(4-1) =9

$E_{22} = (f_{2.} \times f_{.2})/n = (214 \times 175)/\ 495 = 75.6566$

[3] $\chi^2_{22} = (O_{22} - E_{22})^2/E_{22} = (83 - 75.6566)^2/75.6566 = 0.7128$ [1]

The other calculations are done similarly, and $\chi^2_{Calc} = 18.998$ [1]

(1 mark for H0 and Ha, 2 marks for final result—1 mark for some intermediate calculations if final result is incorrect).

b. $X^2_{Crit} = X^2_{0.05}(df = 9) = 16.919$ [1]

[3] Since $\{\chi^2_{Calc} = 18.998\} > \{X^2_{Crit} = 16.919\}$ ➔ Reject H0 [1.5]

Based on the statistical evidence, the distribution of the types of TVs sold is not the same in the various provinces (or The types of TVs sold and the provincial location are not independent variables). [0.5]

(1 mark for critical value, 0.5 mark for conclusion and 0.5 mark for managerial statement)
Plus 1 mark for p-value of approx. 0.025.

c. Since the consumer preferences are different the marketing should be targeted and should be different. {In Alberta, promote HD LCD/LED, in Ontario promote HD 3D etc}.

[1]

d. Since p1 = p3 = p4 = x and 0.5 p2 = p1 ➔ p2 = 2 p1 = 2 x

Thus x + 2 x + x + x = 1 ➔ x = 0.2 ➔

[4] S1: H0: p1 = 0.2, p2 = 0.4, p3 = 0.2, p4 = 0.2

Ha: Not all $p_{i's}$ are as above. [1]

S2: X^2_{Calc} = 7.0909 [1]

S3: X^2_{Crit} = X^2 (df = k – 1 = 3) = 7.8147 [0.5]

S4: Since { X^2_{Calc} = 7.0909} < { X^2_{Crit} = 7.8147} ➔ Do not reject H0 [1]

There is insufficient statistical evidence to claim that the proportions of sales volume are other than p1 = 0.2, p2 = 0.4, p3 = 0.2, p4 = 0.2. [0.5]

Question 3 (13 marks)

a. S1: H0: p = 0.48

[3] Ha: p > 0.48 [0.5]

S2: SD(p-hat) = sqrt{(p0 x q0)/n} = sqrt{(0.48 x 0.52)/300} = 0.0288

p-hat = 155/300 = 0.5167

n x p0 = 144 > 10, and n x q0 = 156 > 10

Z_{Calc}= (p-hat – p0)/SD(p-hat) = (0.5167 – 0.48)/0.0288 = 1.2731 [1.5]

S3: Z_{Crit} = $Z_{(\alpha = 0.05)}$ = 1.645 [0.5]

S4: {Since Z_{Calc} = 1.2731} < {Z_{Crit} = 1.645}➔ Do not reject H0.

Based on the statistical evidence, the proportion is 0.48 and not more than 0.48. [0.5]

b. The appropriate 1 sided CI for a right tail test is LB = p-hat – Z_{Crit} x SE(p-hat)

[2] Here SE(p-hat) = sqrt{(p-hat x q-hat)/n} = sqrt{(0.5167 x 0.4833)/300} = 0.0289 [0.5]

LB = 0.5167 – 1.645 x 0.0289 = 0.5167 – 0.0475 = 0.4692, CI: (0.4692, 1.0) [1]

Since the interval covers p = 0.48, it is consistent with the conclusion reached in part 'a' above. [0.5]

c. Since the test conclusion reached was that 'p' was not more than 0.48, we assume p = 0.48 and since α = 0.05 for a symmetric 95% CI, we find the value of 'n' as follows:

n = $(Z_{\alpha/2}/ME)^2$ x p x q = $(1.96/.015)^2$x0.48x0.52 = 4261.61 or 4262.0 [2]

(1 for correct 'Z' and 1 for correct 'n')

d. S1: H0: p1 – p2 = 0; Ha: p1 – p2 ≠ 0 [0.5]

[4] S2: p_{Pooled} = (X1 + X2)/(n1 + n2) = (155 + 178)/(300 + 400) = (333/700) = 0.4757 [0.5]

SE(p1-hat – p2-hat) = sqrt(p_{Pooled} x q_{Pooled} (1/n1 +1/n2)) = sqrt(0.4757 x 0.5243 (1/300 +1/400))

= 0.0381 [0.5]

Z_{Calc} = {p1-hat – p2-hat - 0}/ SE(p1-hat – p2-hat) = (0.5167 – 0.4450)/0.0381

= 1.8819 [1]

S3: $Z_{Crit} = Z_{\alpha/2} = 1.96$ [0.5]

S4: Since $\{|Z_{Calc}| = 1.8819\} < \{ZCrit = 1.96\}$ → Do not Reject H0. [0.5]

There is insufficient statistical evidence to claim that there is any difference in the proportion of wrong categorization of obese people when using BMI or WHR methodology. [0.5]

e. [2] 95% CI: (p1-hat – p2-hat) ± $Z_{\alpha/2}$ SE(p1-hat – p2-hat)

= 0.0717 ± 1.96 x 0.0381 = 0.0717 ± 0.0747 = (-0.0030, 0.1464) [1.5]

Since this CI covers 'zero', (p1 – p2) could very well be zero and as such it is consistent with the conclusion reached in part 'd' above. [0.5]

Question 4. [24 marks]

a) [1] y-hat = –10.500 – 0.002 X_1 + 0 .001 X_2 + 0.198 X_3 + 1.917 X_4

b) [3]

Source of Variation	D.F.	S.S.	M.S.	F
Regression Model	4	134.538	33.635	27.547
Error	30	36.630	1.221	
Total	34	171.168		

1 mark for D.F., 1 mark for S.S. and 1 mark for M.S. Lose mark for any mistake.

c) [3]

Ho: $\beta_4 = 0$; Ha: $\beta_4 \neq 0$,

t = 1.917/0.225 = 8.52 (df = 35-4-1 = 30); P-value < 0.0005,

Years of education is a useful predictor even for a model that already contains the other three predictors.

1 mark for hypothesis, 1 mark for test statistic, 1 mark for p-value or critical value, 1 mark for conclusion.

d) [2]

For every 1 unit increase in the number of years of education, there is an *estimated average* increase of 1.917 units in wages (not clear what the units of wages are), *assuming the values of the other variables remain constant. (0.5 for "assuming….")*

e) [1]

R-square is 0.786 (value depends on ANOVA table)

f) [3]

Ho: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$; Ha: at least one β_i is not zero

F(0.05,4,30) = 2.69. Since F-stat = 27.547 > 2.69, reject Ho.

The model is useful for explaining variability in wage rate.

1 mark for hypothesis, ½ mark for F-stat, ½ mark for critical value, 1 mark for decision and conclusion.

g) [3] The SE for a prediction interval is sqrt[SE^2(y-hat) + MSE)] = (approx.) sqrt(MSE) = SE of model
y-hat = –1.478+1.607(12) = 17.806
95% PI: 17.806 ± 2(1.529) = 17.806 ± 3.058 or (14.7,20.9)
(Accept multipliers of 1.96 or 2.042 = t(.025,30) instead of 2.

1 mark for y-hat, 1 mark for correct t, 1 mark for correct bounds.

h) [2]
The t-test for age is significant in the original model suggesting that age is a good predictor even in a model that already contains "years of education".
Another sign is that the standard error is lower in the four –variable model.

i) [2]
Calculate the VIF for X4. To do this, regress X4 on the other three variables and find the R-square. The VIF is 1/(1-Rsq). VIF > 10 suggests a potential problem with multicollinearity

j) [2]
-A 2-variable model with age and years of education might be a better model.
-Select this model by dropping the two other variables whose coefficients are not statistically significantly different from zero.

k) [2]
-mention stepwise regression with a brief description, or
-mention all subsets procedure whereby one chooses model based on highest R-sq (or lowest MSE) or some Cp criterion

1 mark for identifying a general method and 1 for describing how it works

Question 5. [14 marks]

(a)
The two key assumptions are that the errors are normally distributed and have constant variance.
The residual plot indicates that the residuals have relatively constant vertical spread and the histogram and NPP of the residuals indicate relative normality.

1 mark for mentioning the two assumptions
1 mark for commenting on the residual plots appropriately.

(b)
The formula is ($3 * 80.67^2 + 3 * 66.09^2 + \ldots + 3 * 87.64^2 + 3 * 194.94^2$) / 18 = 5892

2 marks for showing (weighted) average of sample variances.

(c)
The interaction plot shows there is some interaction since the line segments are not parallel; however, the non-parallelism is not extreme.

Low prices lead to highest sales; High prices lead to lowest sales
Comparatively, not as much difference between the two types of display.

2 marks – 1 for comment on interaction, 1 for comment on main effects.

(d)
Ho: no interaction between Display and Price,
Ha: some interaction between the two factors

F = 2.40 (the p-values are blanked out)

Do not reject Ho since F not > 3.49 (df=2, 20) closest to (df=2,18)
Conclude no interaction

1 for hypotheses
1 for F-stat and critical value of 3.49
1 for decision and conclusion

(e)
The test does not confirm the original observation that there is interaction, but obviously sampling error is causing apparent non-parallelism in the plot of sample means.

1 mark only if reason is clearly stated.
If the answer to (d) actually agrees with (c), then there is nothing to explain, but the correct answer is that they do not agree. In this case, 0.5 mark was awarded. A wrong answer in (c) or (d) and nothing to explain in (e) should not result in an overall mark equal to a right answer in both (c) and (d) when the correct reason (which requires some thought) is not given for (e).

(f)
There are 3 price levels with means = 405.8 740.9 518.4 and J = 3 pairwise comparisons

We need the t-value with tail probability .05/6 = .0083 in each tail for a 2-sided CI.
t(critical) = 2.66 (df = 18 and prob'y = .008) and standard error is 76.7

Since there are 8 observations in each mean, the margin of error is
$\pm$ 2.66 * 76.7 * sqrt(1/8+1/8) = $\pm$ 2.66 * 76.7 / 2 = $\pm$ 102.

CIs are (741 - 406) $\pm$ 102, (518 - 406) $\pm$ 102, (740 - 518) $\pm$ 102.

Since all the pairwise differences exceed 102, they are all significantly nonzero.

4 marks:
1 for t-value of 2.66
1 for standard error of 144
1 for pairwise differences and CIs
1 for conclusion