Concordia
UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	244/4	All
Examination	**Date**	**Pages**
Final	May 2001	2
Instructor		**Course Examiner**
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MARKS

[7] **1.** (a) Find $\delta > 0$ so that if $0 < |x-3| < \delta$ then $|x^2 - 9| < 5$

[7] (b) Show that if $\lim_{x \to a} g(x) = 0$ and $|h(x)| < N$ for all x, then $\lim_{x \to a} g(x)\, h(x) = 0$.

[7] **2.** (a) Show that if f is a function satisfying $|f(x)| \leq \frac{1}{3}|x|$ for all x, then f is continuous at 0.

[7] (b) Show that if f is continuous at a, then $|f|$ is also continuous at a.

[7] **3.** (a) Show that if f is continuous on $[-1, 1]$ such that $x^2 + (f(x))^2 = 1$, then either $f(x) = \sqrt{1-x^2}$ <u>for all x</u>
or $f(x) = -\sqrt{1-x^2}$ <u>for all x</u>.

[7] (b) Show that if f is a continuous function on $[0, 1]$ and $f(x)$ is in $[0, 1]$ for all x, then $f(d) = d$ for some $d \in [0, 1]$.

[7] **4.** (a) Let A, B be 2 nonempty subsets such that $x \leq y$ for all $x \in A, y \in B$.
Show that (i) $\sup A \leq y$ for all $y \in B$
(ii) $\sup A \leq \inf B$

[7] (b) Consider a sequence of closed intervals: $I_1 = [a_1, b_1], I_2 = [a_2, b_2], \ldots$. If $a_n \leq a_{n+1}$ and $b_{n+1} \leq b_n$ for all n, show that there exists a point x in every I_n.

[7] **5.** (a) Suppose f is differentiable at 0 and $f(0) = 0$. Prove that $f(x) = x\, h(x)$ for some function h which is continuous at 0.

[9] (b) Let $f(x) = \begin{cases} h(x) \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

and $h(0) = h'(0) = 0$. Find $f'(0)$.

[7] **6.** (a) If $f'(x) \leq K$ for all x in (a, b), show that $f(b) \leq f(a) + K(b-a)$.

[7] (b) Prove that if $\frac{a_0}{1} + \frac{a_1}{2} + \cdots + \frac{a_n}{n+1} = 0$, then $a_0 + a_1\, x + \cdots + a_n x^n = 0$ for some $x \in [0, 1]$.

[7] **7.** (a) Find g^{-1} in terms of f^{-1} if $g(x) = 5 + f(x)$.

[7] (b) Prove that there is a differentiable function f such that $[f(x)]^7 + f(x) + x = 0$ for all x.