1. (a) Prove that

$$\limsup(a_n + b_n) \le \limsup a_n + \limsup b_n$$

whenever the right-hand-side is not of the form $\infty - \infty$ or $-\infty + \infty$.

(b) Give an example of sequences $\{a_n\}$ and $\{b_n\}$ such that the above inequality is strict.

2. Prove that if $f : \mathcal{R} \to \mathcal{R}$ is continuous at c, then f is bounded on some neighborhood of c.

3. Let $f : [a, b] \to \mathcal{R}$ be continuous. Prove that f is bounded on $[a, b]$.

4. Decide whether the following four statements are TRUE or FALSE. If true, EXPLAIN. If false, provide a COUNTEREXAMPLE.

   (a) All countable sets are well-ordered.

   (b) For $f : R \to \mathcal{R}$, continuity at c implies continuity on a neighborhood of c.

   (c) The function $x - \cos x$ has a solution in the interval $(0, \pi/2)$.

   (d) The sequence $\cos n$ has a convergent subsequence.

5. Let $f : \mathcal{R} \to \mathcal{R}$ be given by

$$f(x) = \begin{cases} x^4 \sin(\frac{5}{x+2x^3}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   Prove that f is differentiable at $x = 0$ and find $f'(0)$.

6. Prove that $f(x) = 1/x$ is not uniformly continuous on $(0, 1]$.

7. (a) Find the Taylor polynomial $P_3(x)$ for the function e^{x^2}, with $x_0 = 0$.

   (b) State the Mean Value Theorem.

   (c) State the Intermediate Value Theorem.