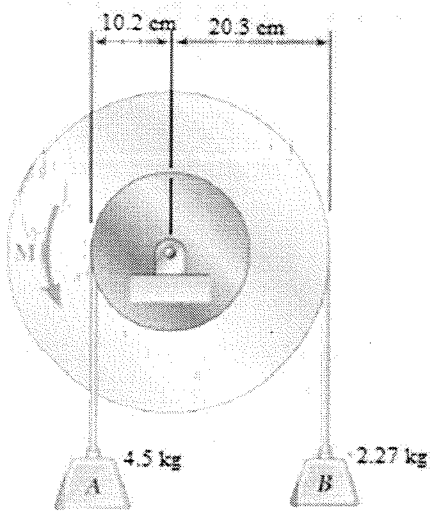
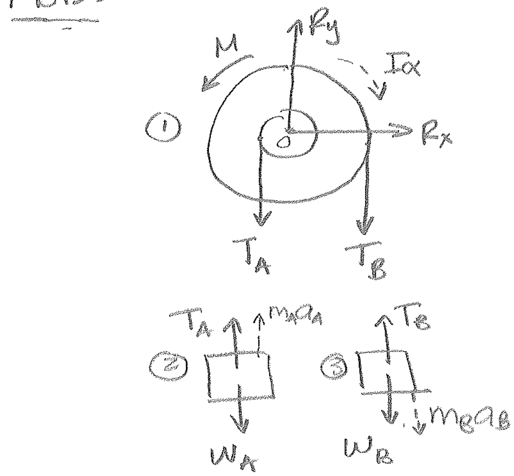
1. The 11.3-kg double pulley shown is at rest and in equilibrium when a constant 4.75 Nm couple M is applied. Neglecting the effect of friction and knowing that the radius of gyration of the double pulley is 15.2 cm, determine (a) the angular acceleration of the double pulley, (b) the tension in each rope.



FBDs



$r_g = 0.152$ m

$I = m_f r_g^2$

$m_f = 11.3$ kg

① $\circlearrowleft + \sum M_o = 0$: $4.75 + T_A(0.102) - T_B(0.203) - I\alpha = 0$

② $+\uparrow \sum F = 0$: $T_A + m_A a_A - W_A = 0$

$T_A = m_A g - m_A a_A$

③ $+\uparrow \sum F = 0$: $T_B = m_B g + m_B a_B$

$4.75 + (m_A g - m_A a_A)(0.102) - (m_B g + m_B a_B)(0.203) - m_f r_g^2 \alpha = 0$

$a_A = r_A \alpha = 0.102\alpha$

$a_B = r_B \alpha = 0.203\alpha$

$4.75 + (m_A g - m_A(0.102\alpha))(0.102) - (m_B g + m_B(0.203\alpha))(0.203) - m_f r_g^2 \alpha = 0$

$4.75 + 4.5 - 0.047\alpha - 4.52 - 0.094\alpha - 0.261\alpha = 0$

$4.73 = 0.402\alpha$

$\boxed{\alpha = 11.77 \text{ rad/s}^2}$

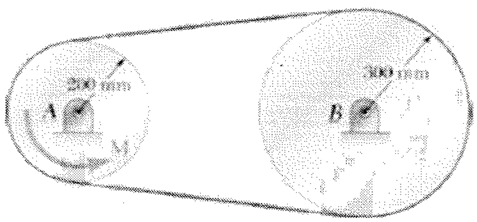
$a_A = 1.20$ m/s²

$a_B = 2.39$ m/s²

From ② and ③ : $T_A = 38.75$ N

$T_B = 27.69$ N

2. Two disks A and B, of mass m_A = 2 kg and m_B = 4 kg, are connected by a belt as shown. Assuming no slipping between the belt and the disks, determine the angular acceleration of each disk if a 2.70 Nm couple M is applied to disk A.

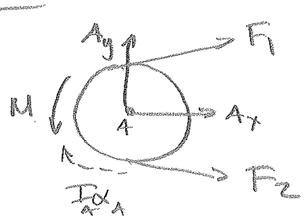


$m_A = 2\,kg$ $\qquad I = \frac{1}{2}mr^2$

$m_B = 4\,kg$

$M_1 = 2.70\,Nm$

Disk A:



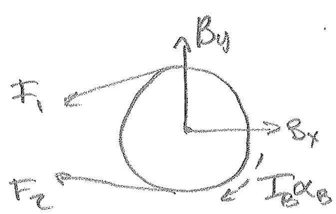
$\circlearrowright + \ \Sigma M_A = 0$

$M + F_2 r_A - F_1 r_A - I_A\alpha_A = 0$

$M + F_2 r_A - F_1 r_A - \frac{1}{2}m_A r_A^2 \alpha_A = 0$

$\frac{M}{r_A} + F_2 - F_1 - \frac{1}{2}m_A r_A \alpha_A = 0$ ——— ①

Disk B:



$\circlearrowright + \ \Sigma M_B = 0$

$F_1 r_B - F_2 r_B - I_B\alpha_B = 0$

$F_1 r_B - F_2 r_B - \frac{1}{2}m_B r_B^2 \alpha_B = 0$

$F_1 - F_2 - \frac{1}{2}m_B r_B \alpha_B = 0$ ——— ②

Add eqns ① and ②

$$\frac{M}{r_A} - \frac{1}{2}m_A r_A \alpha_A - \frac{1}{2}m_B r_B \alpha_B = 0$$

Note that $a_A = a_B$ $\qquad a = r\alpha$

$$r_A\alpha_A = r_B\alpha_B$$

$\frac{M}{r_A} = \frac{1}{2}(m_A + m_B) r_A \alpha_A$ or $\frac{M}{r_B} = \frac{1}{2}(m_A + m_B) r_B \alpha_B$

$\alpha_A = \frac{2M}{r_A^2}\frac{1}{m_A + m_B} = 22.5\ rad/s^2$

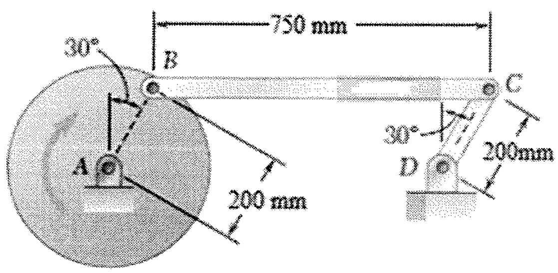
$r_A\alpha_A = r_B\alpha_B \Rightarrow \alpha_B = 15.0\ rad/s^2$

$$\boxed{\begin{array}{l}\alpha_A = 22.5\ rad/s^2 \\ \alpha_B = 15.0\ rad/s^2\end{array}}$$

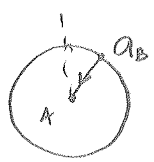
3. The 7.5-kg rod BC connects a disk centered at A to crank CD. Knowing that the disk is made to rotate at the constant speed of 180 rpm, determine for the position shown the vertical component s of the forces exerted on rod BC by the pins at B and C.

$m = 7.5\ kg$

$\omega_A = 180\ rpm = 18.85\ rad/s$



Disk A:



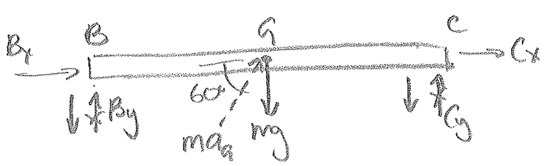
Since $\alpha = 0$, $a_B^t = 0$

$a_B^n = r\omega^2 = (0.2)(18.85)^2$

$a_B^n = 71.061\ m/s^2$

Note that rod BC is translating.

Rod BC



$a_G = a_B^n$ due to translation.

$\circlearrowleft + \Sigma M_B = 0$

$C_y(0.75) - mg(0.375) + ma_G \sin 60(0.375) = 0$

$C_y(0.75) - 7.5(9.81)(0.375) + 7.5(71.061)\sin 60(0.375) = 0$

$C_y = -193.99\ N$

$\boxed{C_y = 193.99\ N \downarrow}$

$+\uparrow \Sigma F_y = 0:$

$B_y - mg + C_y + ma_G \sin 60 = 0$

$B_y - 7.5(9.81) + 7.5(71.061)\sin 60 - 193.99) = 0$

$B_y = -193.99\ N$

$\boxed{B_y = 193.99\ N}$

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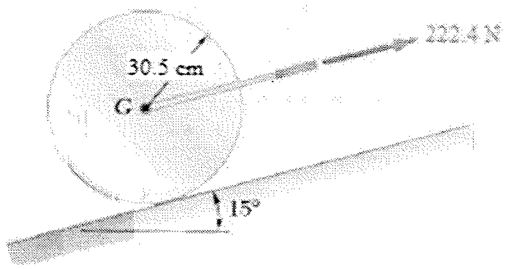
4. A 355.8-N uniform cylinder is acted upon by a 222.4-N force as shown. Knowing that the cylinder rolls without slipping, determine (a) the acceleration of its center G, (b) the minimum value of the coefficient of static friction compatible with this motion.

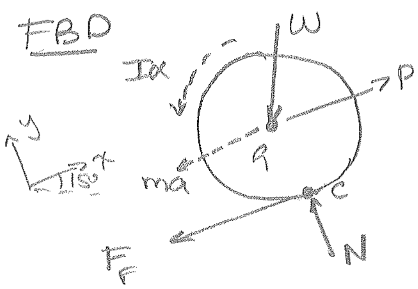


$P = 222.4\,N$

$W = 355.8\,N \rightarrow m = 36.27\,kg$

$r = 30.5\,cm = 0.305\,m$

FBD



$I = \frac{1}{2} m r^2$

$F_f = \mu_s N$, $a = r\alpha$

$+\nearrow \sum F_x = 0$: $P - F_f - ma - W\sin 15 = 0$

$P - \mu_s N - mr\alpha - W\sin 15 = 0$

$222.4 - \mu_s N - 11.06\alpha - 92.09 = 0$

$130.31 - \mu_s N - 11.06\alpha = 0$ ①

$+\uparrow \sum F_y = 0$: $N - W\cos 15 = 0$

$N = 355.8\cos 15$

$N = 343.68\,N$

① $130.31 - 343.68\mu_s - 11.06\alpha = 0$

$+\circlearrowright \sum M_c = 0$: $Pr - W\sin 15 r - mrar - \frac{1}{2} m r^2 \alpha = 0$

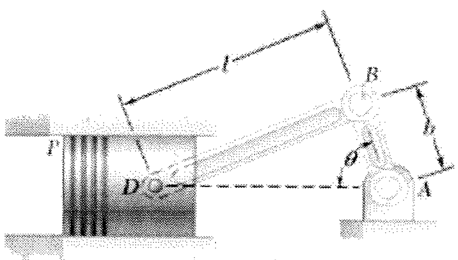
$Pr - W\sin 15 r - mr^2\alpha - \frac{1}{2} m r^2 \alpha = 0$

$222.4(0.305) - 355.8\sin 15(0.305) - \frac{3}{2}(36.27)(0.305)^2\alpha = 0$

$\alpha = 7.85\,rad/s^2 \rightarrow \boxed{a_G = 2.39\,m/s^2 \measuredangle 15°}$

From ①: $\boxed{\mu_s = 0.126}$

5. In the engine system shown l = 25.4 cm and b = 10.2 cm. The connecting rod BD is assumed to be a 1.36 kg uniform slender rod and is attached to the 2-kg piston P. During a test of the system, crank AB is made to rotate with a constant angular velocity of 600 rpm clockwise with no force applied to the face of the piston. Determine the forces exerted on the connecting rod at B and D when θ = 180°. (Neglect the effect of the weight of the rod.)



$l = 25.4\,cm = 0.254\,m = BD$

$b = 10.2\,cm = 0.102\,m = AB$

$m_{BD} = 1.36\,kg$

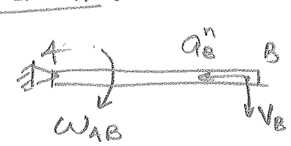
$m_p = 2\,kg$

$\omega_{AB} = 600\,rpm = 62.83\,rad/s$ CW

$\theta = 180°$

Kinematics

AB:

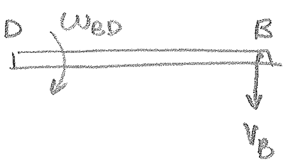


$a_B^t = 0$ since $\alpha = 0$

$\vec{a}_B^n = AB\omega_{AB}^2 = 402.66\,m/s^2 \leftarrow$

$\vec{V}_B = AB\omega = 6.41\,m/s \downarrow$

BD:



$\omega_{BD} = \frac{V_B}{BD} = 25.34\,rad/s$

↳ relates to relative motion

$\overset{x\checkmark}{a_D} = \overset{\checkmark\checkmark}{a_B} + \overset{\checkmark\checkmark}{a_{D/B}}$ (All horizontal)

$\vec{a}_{D/B}^n = 0.254\,\omega_{BD}^2 = 163.1\,m/s^2 \rightarrow$

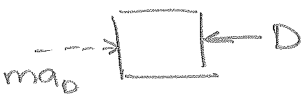
$\vec{a}_D = 239.56\,m/s^2 \leftarrow$

At center of gravity of rod BD : $\bar{a}_{BD} = \frac{1}{2}(a_B + a_D)$

$\bar{a}_{BD} = 321.11\,m/s^2 \leftarrow$

Kinetics

Piston:

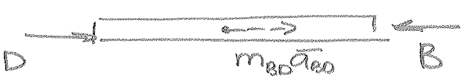


$\xrightarrow{} \Sigma F_x = 0$: $m_p a_D = D$

$D = 2(239.56)$

$\boxed{D = 479.12\ N \longrightarrow}$

Rod BD (Neglect weight)

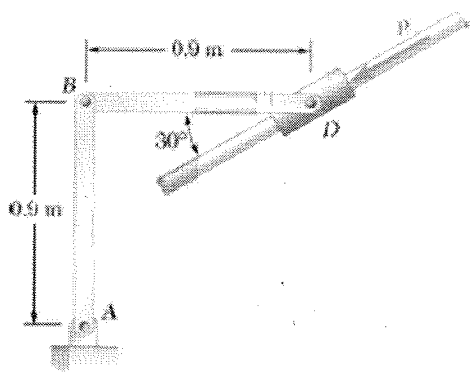


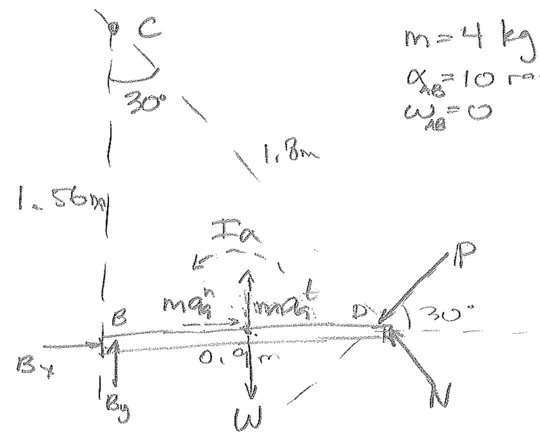
$\overset{+}{\rightarrow} \Sigma F_x = 0$: $D + m_{BD}\bar{a}_{BD} - B = 0$

$479.12 + 1.36(321.11) - B = 0$

$\boxed{B = 915.83\ N \longleftarrow}$

6. The linkage ABD is formed by connecting two 4-kg bars and a collar of negligible mass. The motion of the linkage is controlled by the force P applied to the collar. Knowing that at the instant shown the angular velocity and angular acceleration of bar AB are zero and 10 rad/s² counterclockwise, respectively, determine the force P.





$m = 4\ kg \rightarrow W = 39.24\ N$

$\alpha_{AB} = 10\ rad/s^2$

$\omega_{AB} = 0$

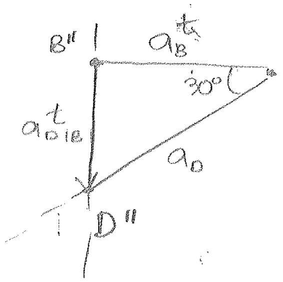
$I\alpha$ direction is assumed.

Kinematics

$a_D = a_B + a_{D/B}$

$a_D = \cancel{a_B^n} + a_B^t + \cancel{a_{D/B}^n} + a_{D/B}^t$

Since $\omega_{AB} = 0$, there is no motion at this instant, therefore $\omega_{BD} = 0$.



$a_B^t = 0.9(10) = 9\ m/s^2 \leftarrow$

$a_{D/B}^t = 0.9\alpha_{BD} \downarrow$

$\frac{a_{D/B}^t}{a_B^t} = \tan 30° \rightarrow a_{D/B}^t = 5.20\ m/s^2$

$\alpha_{BD} = 5.77\ rad/s^2$

$a_G^t = r_{BG}\,\alpha_{BD}$

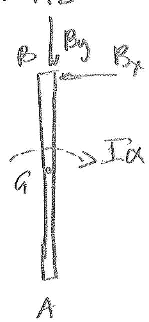
$a_G^t = 0.45(5.77) = 2.60\ m/s^2 \downarrow$

$a_G^n = 9\ m/s^2 \leftarrow$ (Due to a_B^t)

$I = \frac{1}{12} m l^2 = 0.27$

Kinetics

Rod AB



$\circlearrowleft + \ \Sigma M_A = 0$

$B_x(0.9) - I\alpha_{AB} = 0$

$B_x = 30\ N$

Rod BD

$\circlearrowright + \ \Sigma M_C = 0:$

$P(1.8) + W(0.45) - B_x(1.56) + I\alpha_{BD} - m a_G^n(1.56) - m a_G^t(0.45) = 0$

$1.8P + 17.66 - 46.8 - 1.56 - 56.16 - 4.68 = 0$

$\boxed{P = 50.8\ N \ \angle 30^\circ}$