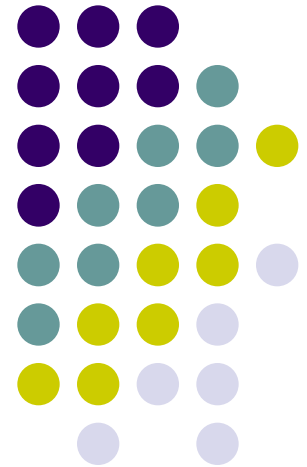
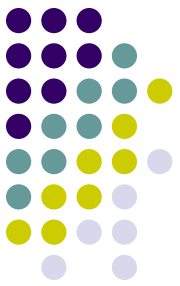


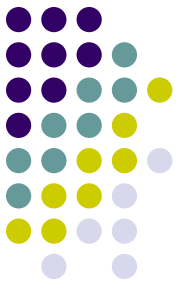
Functional Dependencies





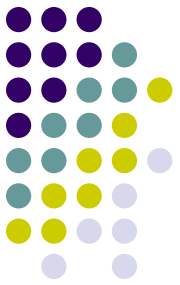
What is FD?

- **Functional dependencies (FDs)** are **constraints** that are derived from the *meaning* and *interrelationships* of the attributes
- A set of attributes X *functionally determines* a set of attributes Y (denoted $X \rightarrow Y$) if **whenever 2 tuples have the same value for X , they *must have* the same value for Y**



Examples of FDs

- Employee's Social Insurance Number determines Employee Name
SIN → EmpName
- Project Number determines Project Name and Project Location
ProjNum → ProjName, ProjLocation
- Employee's Social Insurance Number and Project Number determines the Hours Per Week that the employee works on the project
SIN, ProjNum → HoursPerWeek



Keys & FDs

- K is a **superkey** for relation R if K *functionally determines* all the attributes in R
- K is a **key** for R if K is a **superkey**, but no proper subset of K is a **superkey**.

Sample Data for Drinkers

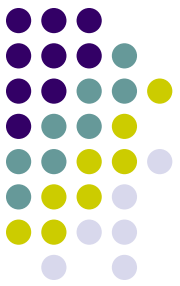


name	addr	beersLiked	manf	favBeer
Zastre	Scott	Buckerfields	Swans	BeaconIPA
Zastre	Scott	BeaconIPA	Beacon	BeaconIPA
Harper	Sussex	Buckerfields	Swans	Buckerfields

name -> addr

name -> favBeer

beersLiked -> manf



Example of Superkey

Drinkers (name, addr, beersLiked, manf, favBeer)

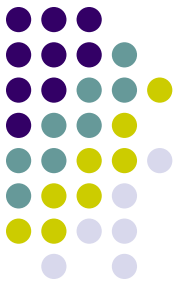
- {name, beersLiked} is a **superkey** because together these attributes determine all the other attributes.
 - name → addr favBeer
 - beersLiked → manf



Example of Key

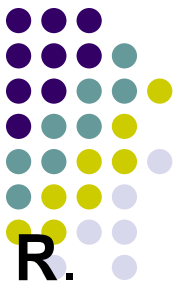
- `{name, beersLiked}` is a **key** because neither `{name}` nor `{beersLiked}` is a **superkey**.
 - `name` does not imply `manf`
 - `beersLiked` does not imply `addr`
- There are no other keys, but lots of superkeys.
 - Any superset of `{name, beersLiked}`

Inference Rules for FDs – 1

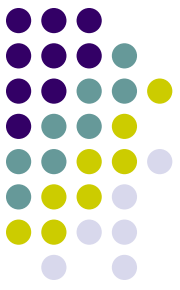


- Given a set of FDs F , we can *infer* additional FDs that hold whenever the FDs in F hold

Inference Rules for FDs



- Let \mathbf{R} be a relation schema, \mathbf{W} , \mathbf{X} , \mathbf{Y} , \mathbf{Z} be subsets of \mathbf{R} .
- **Reflexivity**
 - If $\mathbf{Y} \subseteq \mathbf{X}$, then $\mathbf{X} \rightarrow \mathbf{Y}$ (trivial FD's)
- **Augmentation**
 - If $\mathbf{X} \rightarrow \mathbf{Y}$, then $\mathbf{XZ} \rightarrow \mathbf{YZ}$, for every \mathbf{Z}
- **Transitivity**
 - If $\mathbf{X} \rightarrow \mathbf{Y}$ and $\mathbf{Y} \rightarrow \mathbf{Z}$, then $\mathbf{X} \rightarrow \mathbf{Z}$
- **Union (Combining) Rule**
 - If $\mathbf{X} \rightarrow \mathbf{Y}$ and $\mathbf{X} \rightarrow \mathbf{Z}$, then $\mathbf{X} \rightarrow \mathbf{YZ}$
- **Decomposition (Splitting) Rule**
 - If $\mathbf{X} \rightarrow \mathbf{YZ}$, then $\mathbf{X} \rightarrow \mathbf{Y}$ and $\mathbf{X} \rightarrow \mathbf{Z}$
- **Pseudo-transitivity Rule**
 - If $\mathbf{X} \rightarrow \mathbf{Y}$ and $\mathbf{WY} \rightarrow \mathbf{Z}$, then $\mathbf{XW} \rightarrow \mathbf{Z}$



Question #1

- Consider $R(X, Y, Z, W)$ with FDs $F = \{W \rightarrow Y, X \rightarrow Z\}$
- Prove or disprove $F \models WX \rightarrow Y$

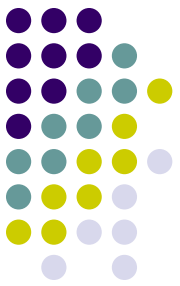
$\{W \rightarrow Y, X \rightarrow Z\}$

$\models \{WX \rightarrow XY\}$ (Augmentation :

If $X \rightarrow Y$, then $XZ \rightarrow YZ$, for every Z)

$\models \{WX \rightarrow Y\}$ (Decomposition Rule:

If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$)



Question #2

- Consider $R(X, Y, Z, W)$ with FDs $F = \{X \rightarrow Y, X \rightarrow W, WY \rightarrow Z\}$
- Prove or disprove $F \models X \rightarrow Z$

$\{X \rightarrow Y, X \rightarrow W, WY \rightarrow Z\}$

$\models \{X \rightarrow WY, WY \rightarrow Z\}$ (**Union Rule:**

If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$)

$\models \{X \rightarrow Z\}$ (**Transitivity:**

If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$)

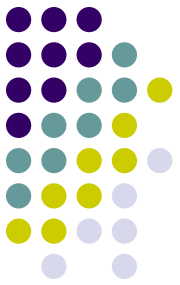


Question #3

- Consider $R(A, B, C)$ with FDs $F = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$
- Prove or disprove $F \models C \rightarrow A$

Counter example:

A	B	C
1	4	3
3	2	3



Closure Test

- A standard way to test if FDs hold is to compute the **closure** of Y , denoted Y^+
 - Note that Y^+ is a set of attributes, not FDs
- **Basis step:** $Y^+ = Y$.
- **Induction:**
 - Look for an FD's left side X that is a subset of the current Y^+
 - If the FD is $X \rightarrow A$, add A to Y^+ .



Question #1 Revisit

- Consider $R(X, Y, Z, W)$ with FDs $F = \{W \rightarrow Y, X \rightarrow Z\}$
- Prove or disprove $F \models WX \rightarrow Y$

$$\mathbf{WX^+ = WXYZ}$$

Since $Y \in WX^+$, $WX \rightarrow Y$ is implied by F

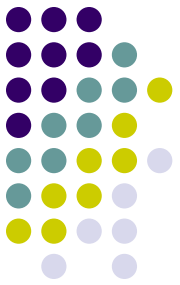


Question #2 Revisit

- Consider $R(X, Y, Z, W)$ with FDs $F = \{X \rightarrow Y, X \rightarrow W, WY \rightarrow Z\}$
- Prove or disprove $F \models X \rightarrow Z$

$$X^+ = XYWZ$$

Since $Z \in X^+$, $X \rightarrow Z$ is implied by F

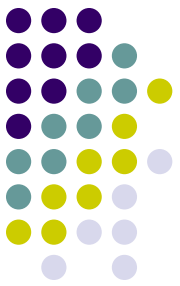


Question #3 Revisit

- Consider $R(A, B, C)$ with FDs $F = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$
- Prove or disprove $F \models C \rightarrow A$

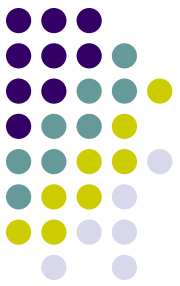
$$C^+ = C$$

Since $A \notin C^+$, $C \rightarrow A$ is not implied by F



Question #4

- Consider a relation with schema $R(A, B, C, D)$ and **FD = $\{AB \rightarrow C, C \rightarrow D, D \rightarrow A\}$**
- *(a) What are all the non-trivial FDs that follow from the given FD's?*
- *(b) What are all the candidate keys of R?*
- *(c) What are all the superkeys of R that are not candidate keys?*



Question #5

- Consider a relation with schema $R(A, B, C, D)$ and **FD = { $AB \rightarrow C$, $BC \rightarrow D$, $CD \rightarrow A$, $AD \rightarrow B$ }**
- *(a) What are all the non-trivial FDs that follow from the given FD's?*
- *(b) What are all the candidate keys of R?*
- *(c) What are all the superkeys of R that are not candidate keys?*