# Functional Dependencies

# What is FD?

- **Functional dependencies** (**FDs**) are **constraints** that are derived from the *meaning* and *interrelationships* of the attributes
- A set of attributes X *functionally determines* a set of attributes Y (denoted X → Y) if whenever 2 tuples have the same value for X, they *must have* the same value for Y

# Examples of FDs

- Employee's Social Insurance Number determines Employee Name

  **SIN → EmpName**
- Project Number determines Project Name and Project Location

  **ProjNum → ProjName, ProjLocation**
- Employee's Social Insurance Number and Project Number determines the Hours Per Week that the employee works on the project

  **SIN, ProjNum → HoursPerWeek**

# Keys & FDs

- *K* is a superkey for relation *R* if *K* *functionally determines* all the attributes in *R*
- *K* is a key for *R* if *K* is a superkey, but no proper subset of *K* is a superkey.

# Sample Data for Drinkers

| name | addr | beersLiked | manf | favBeer |
|---|---|---|---|---|
| Zastre | Scott | Buckerfields | Swans | BeaconIPA |
| Zastre | Scott | BeaconIPA | Beacon | BeaconIPA |
| Harper | Sussex | Buckerfields | Swans | Buckerfields |

name -> addr

name -> favBeer

beersLiked -> manf

# Example of Superkey

`Drinkers(name, addr, beersLiked, manf,favBeer)`

- `{name, beersLiked}` is a superkey because together these attributes determine all the other attributes.
  - `name → addr favBeer`
  - `beersLiked → manf`

# Example of Key

- `{name, beersLiked}` is a key because neither `{name}` nor `{beersLiked}` is a superkey.
  - `name` does not imply `manf`
  - `beersLiked` does not imply `addr`
- There are no other keys, but lots of superkeys.
  - Any superset of `{name, beersLiked}`

# Inference Rules for FDs – 1

- Given a set of FDs F, we can *infer* additional FDs that hold whenever the FDs in F hold

# Inference Rules for FDs

- Let **R** be a relation schema, **W**, **X**, **Y, Z** be subsets of **R**.
- **Reflexivity**
  - If **Y** ⊆ **X**, then **X** → **Y** (trivial FD's)
- **Augmentation**
  - If **X** → **Y**, then **XZ** → **YZ,** for every **Z**
- **Transitivity**
  - If **X** → **Y** and **Y** → **Z**, then **X** → **Z**
- **Union (Combining) Rule**
  - If **X** → **Y** and **X** → **Z**, then **X** → **YZ**
- **Decomposition (Splitting) Rule**
  - If **X** → **YZ**, then **X** → **Y** and **X** → **Z**
- **Pseudo-transitivity Rule**
  - If **X** → **Y** and **WY** → **Z**, then **XW** → **Z**

# Question #1

- Consider R(X, Y, Z, W) with FDs F ={W→Y, X →Z}
- Prove or disprove F |= WX →Y

{W→Y, X →Z}

|= {WX → XY} (Augmentation :

If **X** → **Y**, then **XZ** → **YZ,** for every **Z**)

|= {WX → Y} (Decomposition Rule:

If **X** → **YZ**, then **X** → **Y** and **X** → **Z**)

# Question #2

- Consider R(X, Y, Z, W) with FDs F = $\{X \to Y, X \to W, WY \to Z\}$
- Prove or disprove F $\models X \to Z$

$\{X \to Y, X \to W, WY \to Z\}$

$\models \{X \to WY, WY \to Z\}$ (Union Rule:

If $\mathbf{X} \to \mathbf{Y}$ and $\mathbf{X} \to \mathbf{Z}$, then $\mathbf{X} \to \mathbf{YZ}$)

$\models \{X \to Z\}$ (Transitivity:

If $\mathbf{X} \to \mathbf{Y}$ and $\mathbf{Y} \to \mathbf{Z}$, then $\mathbf{X} \to \mathbf{Z}$)

# Question #3

- Consider R(A, B, C) with FDs F ={A→B, B →C, A→C}
- Prove or disprove F |= C → A

Counter example:

| A | B | C |
|---|---|---|
| 1 | 4 | 3 |
| 3 | 2 | 3 |

# Closure Test

- A standard way to test if FDs hold is to compute the closure of $Y$, denoted $Y^+$
  - Note that $Y^+$ is a set of attributes, not FDs
- Basis step: $Y^+ = Y$.
- Induction:
  - Look for an FD's left side $X$ that is a subset of the current $Y^+$
  - If the FD is $X$ -> $A$, add $A$ to $Y^+$.

# Question #1 Revisit

- Consider R(X, Y, Z, W) with FDs F ={$W \rightarrow Y$, $X \rightarrow Z$}
- Prove or disprove F |= $WX \rightarrow Y$

**$WX^{+}$ = WXYZ**

Since $Y \in WX^{+}$, $WX \rightarrow Y$ is implied by F

# Question #2 Revisit

- Consider R(X, Y, Z, W) with FDs F ={$X \rightarrow Y$, $X \rightarrow W$, $WY \rightarrow Z$}
- Prove or disprove F |= $X \rightarrow Z$

**$X^+$ = XYWZ**

Since $Z \in X^+$, $X \rightarrow Z$ is implied by F

# Question #3 Revisit

- Consider R(A, B, C) with FDs F ={A→B, B →C, A→C}
- Prove or disprove F |= C → A

**$C^+ = C$**

Since $A \notin C^+$, C →A is not implied by F

# Question #4

- Consider a relation with schema R(A, B, C, D) and **FD = {AB → C, C → D, D → A}**
- *(a) What are all the non-trivial FDs that follow from the given FD's?*
- *(b) What are all the candidate keys of R?*
- *(c) What are all the superkeys of R that are not candidate keys?*

# Question #5

- Consider a relation with schema R(A, B, C, D) and **FD = {AB → C, BC → D, CD → A, AD→B}**
- *(a) What are all the non-trivial FDs that follow from the given FD's?*
- *(b) What are all the candidate keys of R?*
- *(c) What are all the superkeys of R that are not candidate keys?*