

MATH 3705A
Test 1 Solutions
January 23, 2009

[Marks] Questions 1-4 are multiple choice. Circle the correct answer. Only the answer will be marked.

[3] 1. $\mathcal{L}\{e^{2t} \cos(3t)\} =$
(a) $\frac{s}{(s-2)^2+9}$ (b) $\frac{s-2}{s^2+9}$ (c) $\frac{s-2}{(s-2)^2+9}$ (d) $\frac{s+2}{(s+2)^2+9}$ (e) None of these

Solution: (c)

[3] 2. $\mathcal{L}\{u(t-3)e^{-2t}\} =$
(a) $\frac{e^{-3(s+2)}}{s+2}$ (b) $\frac{e^{-3s}}{s+2}$ (c) $\frac{e^{3s}}{s+2}$ (d) $\frac{e^{-3(s-2)}}{s+2}$ (e) None of these

Solution: (a)

[3] 3. $\mathcal{L}^{-1}\left\{\frac{6s}{(s^2+9)^2}\right\} =$
(a) $t \sin(3t)$ (b) $t \cos(3t)$ (c) $-t \sin(3t)$ (d) $-t \cos(3t)$ (e) None of these

Solution: (a)

[3] 4. $\mathcal{L}^{-1}\left\{\frac{se^{-s}}{s^2-2s+4}\right\} =$
(a) $e^t \cos(\sqrt{3}t) + \frac{1}{3}e^{3t} \sin(\sqrt{3}t)$
(b) $u(t-1)\{e^{t-1} \cos[\sqrt{3}(t-1)] + e^{t-1} \sin[\sqrt{3}(t-1)]\}$
(c) $u(t-1)e^{t-1}\{\cos[\sqrt{3}(t-1)] + \frac{1}{\sqrt{3}} \sin[\sqrt{3}(t-1)]\}$
(d) $u(t-1)\{e^{t-1} \cos[\sqrt{3}(t-1)] + \frac{1}{3}e^{t-1} \sin[\sqrt{3}(t-1)]\}$
(e) None of the above

Solution: (c)

[9] 5. Solve the initial-value problem $y'' - y' + 6y = 0$, $y(0) = 0$, $y'(0) = 5$.

Solution:

$$[s^2Y(s) - sy(0) - y'(0)] - [sY(s) - y(0)] + 6Y(s) = 0 \Rightarrow$$
$$(s^2 - s + 6)Y(s) - 5 = 0 \Rightarrow Y(s) = \frac{5}{s^2 - s + 6} = \frac{5}{(s - 1/2)^2 + 23/4} \Rightarrow$$

$$y(t) = \frac{10}{\sqrt{23}} e^{\frac{1}{2}t} \sin\left(\frac{\sqrt{23}}{2}t\right).$$

For those who guessed correctly that the intended equation was $y'' - y' - 6y = 0$, the solution is below.

$$[s^2Y(s) - sy(0) - y'(0)] - [sY(s) - y(0)] - 6Y(s) = 0 \Rightarrow$$

$$(s^2 - s - 6)Y(s) - 5 = 0 \Rightarrow Y(s) = \frac{5}{s^2 - s - 6} = \frac{5}{(s+2)(s-3)} = \left[\frac{1}{s-3} - \frac{1}{s+2} \right]$$

by partial fractions, hence $y(t) = (e^{3t} - e^{-2t})$.

- [9] 6. Solve the initial-value problem $y'' + 4y' + 13y = 0$, $y(0) = 2$, $y'(0) = 4$.

Solution:

$$[s^2Y(s) - sy(0) - y'(0)] + 4[sY(s) - y(0)] + 13Y(s) = 0 \Rightarrow$$

$$(s^2 + 4s + 13)Y(s) - 2s - 12 = 0 \Rightarrow Y(s) = 2 \frac{s+6}{s^2 + 4s + 13} = 2 \frac{(s+2) + 4}{(s+2)^2 + 9} =$$

$$2 \frac{s+2}{(s+2)^2 + 9} + \frac{8}{3} \frac{3}{(s+2)^2 + 9} \Rightarrow y(t) = 2e^{-2t} \cos(3t) + \frac{8}{3} e^{-2t} \sin(3t).$$