**MAST-224, Winter 2013**
**The Big M Method, Bevco LPP, Dorian LPP**

**The Simplex Method requires a starting BFS (IBFS). In all problems up to now we have found starting BFS (IBFS) by using the slack variables as our basic variables.**
However, if a given LPP has $\geq$ or $=$ constraints a starting BFS may not be obvious. In such a case, there are methods, for example the Big M Method (a modification of the Simplex Method), that can be used to find easily an initial basic feasible solution (IBFS) and with this IBFS (BFS0) to start the Simplex Method in order to solve the problem.

We shall explain the Big M Method by using a real life example:
Bevco Oranj Production.

**Real Life Problem.** Bevco Oranj Production manufactures an orange soft drink called Oranj by blending (mixing) **orange soda** and **orange juice**.
- each ounce of orange soda contains 0.5 oz of sugar and 1 mg of vitamin C;
- each ounce of orange juice contains 0.25 oz of sugar and 3 mg of vitamin C;
- it costs Bevco 2 c to produce an ounce of orange soda and 3 c to produce an ounce of orange juice.

Bevco is manufacturing 10 ounces bottles of Oranj. Bevco's Marketing Dept. has decided that each 10 ounces bottle of Oranj must contain **at least** 20 mg of vitamin C and **at most** 4 oz of sugar. By using an appropriate LPP, determine how Bevco can meet the Marketing Dept's requirements at a MIN cost.
**Solution.** Denote by $x_1$ the # of ounces of orange soda in a bottle and with $x_2$ the # of ounces of orange juice in a bottle. Then the LPP corresponding to the Bevco Oranj Production is the following:

$$\begin{aligned}
&\min z = 2x_1 + 3x_2\\
&\text{s.t.}\\
&0.5x_1 + 0.25x_2 \leq 4\\
&x_1 + 3x_2 \geq 20\\
&x_1 + x_2 = 10\\
&x_1,\ x_2 \geq 0
\end{aligned}$$

The LPP is in standard form. Now we put the LPP in Canonical Form by using slack and excess variables (it is convenient to number your excess and slack variables with indexes corresponding to the number of the corresponding Rows):

$$\min z = 2x_1 + 3x_2$$
s.t.
$$0.5x_1 + 0.25x_2 + s_1 = 4$$
$$x_1 + 3x_2 - e_2 = 20$$
$$x_1 + x_2 = 10$$
$$x_1,\ x_2 \geq 0$$

**To start the Simplex Algorithm an initial BFS (IBFS) is needed. We shall create artificial variables $a_i$ for $\geq$ and $=$ constraints,i.e., in the Rows of the Canonical Form where we have excess variables; or neither excess nor slack variables. In our LP Problem these are the Rows 2 and 3.**
**It is convenient to number the artificial variables with indexes corresponding to the number of the Rows where they have been entered.**

$$\min z = 2x_1 + 3x_2 \qquad (1)$$
s.t.
$$0.5x_1 + 0.25x_2 + s_1 = 4$$
$$x_1 + 3x_2 - e_2 + a_2 = 20$$
$$x_1 + x_2 + a_3 = 10$$
$$x_1,\ x_2 \geq 0$$

**Here is the Idea of The Big M Method: In the optimal solution all artificial variables must be set to zero in order the optimal solution to correspond to our given LPP, without artificial variables. To achieve this effect we change the objective function as it follows. Let M be a very large positive number, M >> 0.**

–for a MAX LPP a term $-Ma_i$ is added to the right hand-side of the objective function, or equivalently, a term $Ma_i$ is added to the left hand-side which is the same.
–for a MIN LPP a term $Ma_i$ is added to the right hand-side of the objective function, or equivalently, a term $-Ma_i$ is added to the left hand-side, which is the same.

In our MIN LPP we have to add $Ma_2$ and $Ma_3$ to the right hand-side of the objective function (1), or equivalently, the terms $-Ma_2$ and $-Ma_3$ can be added to the left hand-side of the objective function.

**Bevco LPP is a MIN LP hence the Row 0 will be changed as it follows:**

$$z = 2x_1 + 3x_2 + Ma_1 + Ma_2 \quad \Leftrightarrow \quad z - 2x_1 - 3x_2 - Ma_1 - Ma_2 = 0$$

Bevco LPP is a MIN LPP hence, we have the following: **Big M Preliminary Form:**

$$\begin{aligned} Row0: &\quad z - 2x_1 - 3x_2 - Ma_2 - Ma_3 = 0 \\ Row1: &\quad 0.5x_1 + 0.25x_2 + s_1 = 4 \\ Row2: &\quad x_1 + 3x_2 - e_2 + a_2 = 20 \\ Row3: &\quad x_1 + x_2 + a_3 = 10 \end{aligned}$$

**Idea: Given a MIN LP. If M $>>$ 0 very large positive number then, if $a_2 > 0$ or $a_3 > 0$ makes z to be extremely big so, the optimal min solution will force $a_2 = a_3 = 0$ when $M$ is very large positive number.**
**What about MAX LP? Given a MAX LP. If M $>>$ 0 very large positive number then, if $a_2 > 0$ or $a_3 > 0$ makes z to be extremely small negative so, the optimal max solution will force $a_2 = a_3 = 0$ when M is very large positive number.**

Here are the basic steps of the Big M Method that is a modification of the Simplex Method:

1. Modify the constraints so that the r.h.s. of each constraint to be non-negative and all variables $\geq 0$ by sign (Standard Form of the given LPP)).

2. Put the LPP in Canonical Form by using slack $s_i$ and excess $e_i$ variables.

3. For each constraint $\geq$ or $=$ in the Standard Form of the LPP or in other words for each constraint in the Canonical Form containing excess variable or having (neither excess nor slack variable) add an artificial variables $a_i$.

4. All decision, slack, excess, and artificial variables must satisfy the sign simplex restriction $\geq 0$.

5. For MIN LPP: Add $Ma_i$ **on the right hand-side** in the objective function for each artificial variable $a_i$.
For MAX LPP: Add $-Ma_i$ **on the right hand side** in the objective function for each artificial variable $a_i$.

6. Since each of the artificial variables will be a BV (basic variable) in the starting (initial) BFS (IBFS): Then all artificial variables must be eliminated from Row 0 in order to obtain Row 0 of the Canonical Tableau 0 of the Big M Method.

7. Solve the Big M LPP starting with the Canonical Tableau 0 by using the Simplex Method.

**At the end of your solution, in other words in the Optimal Canonical Tableau two cases are possible:**

**(a) If in the Optimal Canonical Tableau all artificial variables are equal to** 0**, the solution is optimal for the given LP.**

**(b) If at least one artificial variable is positive in the optimal solution, the given LPP is infeasible (no feasible region for our starting (given)**

**LPP).**

**Summing up, the Big M Method is a modification of the Simplex Method in case of excess constraints or equality (neither excess nor slack) constraints in the corresponding Canonical Form.**

Here is the solution of Bevco LP starting with the Big M Tableau 0:

**Canonical Tableau 0:**

| $z$ | $x_1$ | $\mathbf{x}_2$ | $s_1$ | $e_2$ | $a_2$ | $a_3$ | r.h.s. | BV | r.t. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | $2M-2$ | $4M-3$ | 0 | $-M$ | 0 | 0 | $30M$ | $z=30M$ | |
| 0 | 1/2 | 1/4 | 1 | 0 | 0 | 0 | 4 | $s_1=4$ | 16 |
| 0 | 1 | 3 | 0 | −1 | 1 | 0 | 20 | $a_2=20$ | $20/3^*$ |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 10 | $a_3=10$ | 10 |

The r.t. gives that $x_2$ will enter as BV ($x_2$ will enter the basis) replacing $a_2$ in Row 2.

**Canonical Tableau 1:**

| $z$ | $\mathbf{x}_1$ | $x_2$ | $s_1$ | $e_2$ | $a_2$ | $a_3$ | r.h.s. | BV | r.t. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | $\frac{2M-3}{3}$ | 0 | 0 | $\frac{M-3}{3}$ | $\frac{3-4M}{3}$ | 0 | $20+\frac{10M}{3}$ | $z=20+\frac{10M}{3}$ | |
| 0 | 5/12 | 0 | 1 | 1/12 | −1/12 | 0 | 7/3 | $s_1=7/3$ | 28/5 |
| 0 | 1/3 | 1 | 0 | −1/3 | 1/3 | 0 | 20/3 | $x_2=20/3$ | 20 |
| 0 | 2/3 | 0 | 0 | 1/3 | −1/3 | 1 | 10/3 | $a_3=10/3$ | $5^*$ |

The r.t. gives that $x_1$ will enter as BV replacing $a_3$ in Row 3.

**Canonical Tableau 2:**

| $z$ | $x_1$ | $x_2$ | $s_1$ | $e_2$ | $a_2$ | $a_3$ | r.h.s. | BV | r.t. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | −1/2 | $\frac{1-2M}{2}$ | $\frac{3-2M}{2}$ | 25 | $z=25$ | |
| 0 | 0 | 0 | 1 | −1/8 | 1/8 | −5/8 | 1/4 | $s_1=1/4$ | |
| 0 | 0 | 1 | 0 | −1/2 | 1/2 | −1/2 | 5 | $x_2=5$ | |
| 0 | 1 | 0 | 0 | 1/2 | −1/2 | 3/2 | 5 | $x_1=5$ | |

**Canonical Tableau 2:**

| $z$ | $x_1$ | $x_2$ | $s_1$ | $e_2$ | $a_2$ | $a_3$ | r.h.s. | BV | r.t. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | $-1/2$ | $\frac{1-2M}{2}$ | $\frac{3-2M}{2}$ | 25 | $z = 25$ | |
| 0 | 0 | 0 | 1 | $-1/8$ | $1/8$ | $-5/8$ | $1/4$ | $s_1 = 1/4$ | |
| 0 | 0 | 1 | 0 | $-1/2$ | $1/2$ | $-1/2$ | 5 | $x_2 = 5$ | |
| 0 | 1 | 0 | 0 | $1/2$ | $-1/2$ | $3/2$ | 5 | $x_1 = 5$ | |

Solving Row 0 with respect to $z$ we obtain:

$$z = \frac{1}{2}e_2 + \frac{2M-1}{2}a_2 + \frac{2M-3}{2}a_3 + 25$$

to conclude that **the above Canonical Tableau 2 is optimal.** This can be seen from the Canonical Tableau 2 directly, taking into account that all NBV in Row 0 of Canonical Tableau 2 are with negative coefficients. Any attempt to enter one of these variable as BV will increase the value of the objective function but we a looking for MIN. **Note that M is a very big positive number, $M >> 0$.**

So, an optimal BFS for the Big M form of our LPP is:

$$BFS\ \ BV:\ z_{min} = 25, x_1 = 5, x_2 = 5, s_1 = 1/4;$$

$$NBV:\ e_2 = 0, a_2 = 0, a_3 = 0\,.$$

**The optimal BFS of the Big M LPP is unique because all coefficients of the NBV in Row 0 of the Optimal Tableau 2 are distinct from 0.**

This gives a unique optimal BFS for the Canonical Form of the Bevco LPP (after truncation of the artificial variables):

$$z_{min} = 25, x_1 = 5, x_2 = 5, s_1 = 1/4, e_2 = 0\,.$$

Now, truncating the slack and the excess variables will give an optimal solution of the given LPP. Hence, **the unique optimal solution of the Bevco LP is:**

$$z_{min} = 25, x_1 = 5, x_2 = 5\ .$$

**Remark.** The given LPP is a two-variable LPP and we can solve it graphically, also. The feasible region is a line-segment on the $(x_1, x_2)$-plane, obviously bounded so, you have to take only the value of the goal functions $z$ at the two end-points of **the line-segment feasible region**.

## Solution of Dorian Auto LPP by the Big M Method

**Problem.** Use the Big M Method to solve Dorian Auto LPP.

**Solution.** We start with the Mathematical LPP Model of the Dorian Auto

$$\begin{aligned}
&\min z = 50x_1 + 100x_2 \\
&\text{s.t.} \\
&7x_1 + 2x_2 \geq 28 \\
&2x_1 + 12x_2 \geq 24 \\
&x_1,\ x_2 \geq 0
\end{aligned}$$

Now we change our problem in Canonical Form by using slack and excess variables (always number your excess and slack variables with indexes corresponding to the number of the Row):

$$\begin{aligned}
&\min z = 50x_1 + 100x_2 \\
&\text{s.t.} \\
&7x_1 + 2x_2 - e_1 = 28 \\
&2x_1 + 12x_2 - e_2 = 24 \\
&x_1,\ x_2,\ e_1,\ e_2 \geq 0
\end{aligned}$$

In order to start the Simplex Algorithm an initial BFS (IBFS) is needed. We shall create **artificial variables $a_i$ for $\geq$ and $=$ constraints, i.e., in the Rows of the Canonical Form where we do have**
**–either excess variables;**
**–or neither excess nor slack variables.**
In our Problem Rows 1 and 2 contain excess variables so, we have to add two artificial variables $a_1$ and $a_2$. Always number the artificial variables with indexes corresponding to the number of the Row where they have been entered.**We have in Row 1 and in Row 2 excess variables so we introduce the artificial variables $a_1$ for Row 1 and $a_2$ for Row 2.**

$$\begin{aligned}
&\min z = 50x_1 + 100x_2 \qquad (1) \\
&\text{s.t.} \\
&7x_1 + 2x_2 - e_1 + a_1 = 28 \\
&2x_1 + 12x_2 - e_2 + a_2 = 24 \\
&x_1,\ x_2, e_1, e_2, a_1, a_2 \geq 0
\end{aligned}$$

**In the optimal solution all artificial variables must be set to zero. To achieve this effect we change the objective function as it follows: Let $M$ be a very large positive number. We have a MIN LPP so, $Ma_1$ and $Ma_2$ will be added to the right hand side of (1).**

The Dorian Auto LPP will have the following **Big M preliminary form. Why preliminary? Because in Row 0 we have the BV $a_1$ and $a_2$ so, they must be excluded from Row 0 by using ERO**:

$$\begin{array}{ll} Row0: & z - 50x_1 - 100x_2 - Ma_1 - Ma_2 = 0 \\ Row1: & 7x_1 + 2x_2 - e_1 + a_1 = 28 \\ Row2: & 2x_1 + 12x_2 - e_2 + a_2 = 24 \\ x_1,\ x_2, e_1, e_2, a_1, a_2 \geq 0 & \end{array}$$

If $M >>$ very large positive number then $a_1 > 0$ and $a_2 > 0$ makes $z$ to be extremely costly so, the optimal solution force $a_1 = a_2 = 0$.
Let us repeat the basic steps of the Big M Method:

1. Modify the constraints so that the RHS of each constraint to be non-negative and all variables to be $\geq 0$ by sign (Standard Form).
2. Canonical Form form by using slack $s_i$ and excess $e_i$ variables.
3. For each constraint $\geq$ or $=$ or in other words for each constraint in the Canonical Form containing excess variable or having neither excess nor slack variable add an artificial variable $a_i$.
4. All decision, slack, excess, and artificial variables must satisfy the sign restriction $\geq 0$.
5. Add terms $Ma_i$ in the objective function for MIN LPP on the right hand-side of Row 0. Add terms $-Ma_i$ in the objective function for MAX LPP on the right hand-side of Row 0.
6. Since each of the artificial variables will be a BV in the starting (initial) BFS (IBFS) they must be eliminated from Row 0 in order to obtain Row 0 of the Canonical Tableau 0.

In our case we have:

$$\begin{array}{ll} Row\ 0: & z - 50x_1 - 100x_2 - Ma_1 - Ma_2 = 0 \\ Row\ 1: & 7x_1 + 2x_2 - e_1 + a_1 = 28 \\ Row\ 2: & 2x_1 + 12x_2 - e_2 + a_2 = 24 \end{array}$$

and after performing the elementary row operation:

$$Row0 \leftarrow Row0 + M \times Row1 + M \times Row2$$

we obtain the $Row0$ of the Big M Canonical Tableau 0:

**Canonical Tableau 0:**

| $z$ | $x_1$ | $\mathbf{x}_2$ | $e_1$ | $e_2$ | $a_2$ | $a_3$ | rhs | BV | r.t. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | $9M - 50$ | $14M - 100$ | $-M$ | $-M$ | 0 | 0 | $52M$ | $z = 52M$ | |
| 0 | 7 | 2 | $-1$ | 0 | 1 | 0 | 28 | $a_1 = 28$ | 14 |
| 0 | 2 | 12 | 0 | $-1$ | 0 | 1 | 24 | $a_2 = 24$ | $2^*$ |

7. Solve the transformed LPP by using the Simplex Method starting with the Canonical Tableau 0. **At the end of the solution, in other words in the Optimal Canonical Tableau two cases are possible:**
**(a)** If in the Optimal Tableau all artificial variables are equal to 0, the solution is optimal for the given LP.
**(b)** If at least one artificial variable is positive in the optimal solution, the LP is infeasible (no feasible region for our starting LP).
Here is the solution of Dorian Auto LPP starting with the Big M Tableau 0:

**Tableau 0:**

| $z$ | $x_1$ | $\mathbf{x}_2$ | $e_1$ | $e_2$ | $a_1$ | $a_2$ | rhs | BV | r.t. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | $9M-50$ | $14M-100$ | $-M$ | $-M$ | 0 | 0 | $52M$ | $z = 52M$ | |
| 0 | 7 | 2 | $-1$ | 0 | 1 | 0 | 28 | $a_1 = 28$ | 14 |
| 0 | 2 | 12 | 0 | $-1$ | 0 | 1 | 24 | $a_2 = 24$ | $2^*$ |

The r.t. gives that $x_2$ will enter as BV replacing $a_2$ in Row 2.

**Tableau 1:**

| $z$ | $\mathbf{x}_1$ | $x_2$ | $e_1$ | $e_2$ | $a_1$ | $a_2$ | rhs | BV | r.t. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | $\frac{20M-100}{3}$ | 0 | $-M$ | $\frac{M-50}{6}$ | 0 | $\frac{50-7M}{6}$ | $24M+200$ | $z = 24M+200$ | |
| 0 | $20/3$ | 0 | $-1$ | $1/6$ | 1 | $-1/6$ | 24 | $a_1 = 24$ | $\frac{18}{5}^*$ |
| 0 | $1/6$ | 1 | 0 | $-1/12$ | 0 | $1/12$ | 2 | $x_2 = 2$ | 12 |

The r.t. gives that $x_1$ will enter as BV replacing $a_1$ in Row 1.

**Tableau 2:**

| $z$ | $\mathbf{x}_1$ | $x_2$ | $e_1$ | $e_2$ | $a_1$ | $a_2$ | rhs | BV | r.t. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | $-5$ | $-7.5$ | $5-M$ | $\frac{55-8M}{6}$ | 320 | $z = 320$ | |
| 0 | 1 | 0 | $-3/20$ | $1/40$ | $3/20$ | $-1/40$ | $18/5$ | $x_1 = 18/5 = 3.6$ | |
| 0 | 0 | 1 | $1/40$ | $-21/240$ | $-1/40$ | $21/240$ | $7/5$ | $x_2 = 7/5 = 1.4$ | |

The above Tableau is optimal because all NBV in Row 0 are with negative coefficients. We can see this also by solving Row 0 with respect to $z$:

$$z = 5e_1 + 7.5e_2 + (M-5)a_1 + \frac{8M-55}{6}a_2 + 320\,.$$

Any attempt to enter one of the NBV variables on the right as a BV will increase the value of the objective function but we are looking for MIN. Hence, the unique (all coefficients in front of the NBV in Row 0 are nonzero) **optimal BFS for the Big M form of our LPP is**:

$z_{min} = 320$, BV: $x_1 = 3.6$, $x_2 = 1.4$; NBV: $e_1 = 0, e_2 = 0, a_1 = 0, a_2 = 0$ which gives a unique optimal BFS for the Canonical LPP of Dorian Auto:

$$z_{min} = 320, x_1 = 3.6, x_2 = 1.4, e_1 = 0, e_2 = 0\,.$$

Hence, $z_{min} = 320, x_1 = 3.6, x_2 = 1.4$ is the unique optimal solution of the initial Dorian Auto LPP.

**Remark.** As we know the problem can be solved graphically and the graphical solution also shows that the Dorian LPP has a unique optimal solution.

## Spot an Infeasible LPP by the Big M Method
## Bevco Modified Example

The Simplex Method requires a Starting BFS In all problems up to now we have found starting BFS by using the slack variables as our basic variables. However, if LPP has $\geq$ or $=$ constraints a starting BFS may be not obvious. In such a case, there are methods, for example the Big M Method, that can be used to find an initial BFS and to solve the problem. In the previous material we explained the Big M Method on a concrete example: Bevco Oranj production.

**Here we show how to Spot an Infeasible LPP by making use of the Big M Method. We modify the Bevco problem by requiring the 10 ounces bottle of Oranj produced by Bevco, to contain at least 31 mg of Vitamin C.**

**One 10 ounces bottle of Oranj can contain at most** $10 \times 3 = 30$ **mg vitamin C so, we know at the very beginning that our problem is infeasible. Let us see how the Big M Method will spot (show, reveal) an infeasible LPP.**

**Bevco manufacturing modified:**

Bevco LPP manufactures an orange soft drink called Oranj by blending (mixing) **orange soda** and **orange juice**.

each ounce of orange soda contains 0.5 oz of sugar and 1 mg of vitamin C;
each ounce of orange juice contains 0.25 oz of sugar and 3 mg of vitamin C;
it costs Bevco 2 c to produce an ounce of orange soda and 3 c to produce an ounce of orange juice

Bevco is manufacturing 10 ounces bottles of Oranj. Bevco's Marketing Dept. has decided that each 10 ounces bottle of Oranj must contain **at least** 31 mg of vitamin C and **at most** 4 oz of sugar.

**Problem.** Use LPP to determine how Bevco can meet the Marketing Dept's requirements at a MIN cost.

**Solution.** Denote by $x_1$ the # of ounces of orange soda in a bottle and with $x_2$ the # of ounces of orange juice in a bottle. Then the Mathematical Model or the

LPP corresponding to the Bevco case is the following:

$$\begin{aligned}
&\min z = 2x_1 + 3x_2\\
&\text{s.t.}\\
&0.5x_1 + 0.25x_2 \leq 4\\
&x_1 + 3x_2 \geq 31\\
&x_1 + x_2 = 10\\
&x_1,\ x_2 \geq 0
\end{aligned}$$

Now we change our problem in Canonical Form (CLPP) by using slack and excess variables (always number your excess and slack variables with indexes corresponding to the number of the Row where they have been entered):

$$\begin{aligned}
&\min z = 2x_1 + 3x_2\\
&\text{s.t.}\\
&0.5x_1 + 0.25x_2 + s_1 = 4\\
&x_1 + 3x_2 - e_2 = 31\\
&x_1 + x_2 = 10\\
&x_1,\ x_2 \geq 0
\end{aligned}$$

Now, in order to start the Simplex Algorithm an initial BFS is needed. We shall create **artificial variables** $a_i$ for $\geq$ and $=$ constraints,i.e., **in the Rows of the Standard Form where we do have excess variables ; or neither excess nor slack variables. In our Problem these are Rows 2 and 3.**
**Always number the artificial variables with indexes corresponding to the number of the Row where they have been entered.**

$$\begin{aligned}
&\min z = 2x_1 + 3x_2 \qquad (1)\\
&\text{s.t.}\\
&0.5x_1 + 0.25x_2 + s_1 = 4\\
&x_1 + 3x_2 - e_2 + a_2 = 31\\
&x_1 + x_2 + a_3 = 10\\
&x_1,\ x_2 \geq 0
\end{aligned}$$

**Now we follow the idea of The Big M Method: In the optimal solution all artificial variables must be set to zero. To achieve (to accomplish) (to obtain) this effect we change the objective function as it follows: Let $M >> 0$ be a very large positive number. We have a MIN problem so, terms $Ma_i$ is to be added to the right hand side of (1).**

So, the Bevco Modified LPP will have the following: **Big M Preliminary Form**:

$$\begin{array}{ll} Row0: & z - 2x_1 - 3x_2 - Ma_2 - Ma_3 = 0 \\ Row1: & 0.5x_1 + 0.25x_2 + s_1 = 4 \\ Row2: & x_1 + 3x_2 - e_2 + a_2 = 31 \\ Row3: & x_1 + x_2 + a_3 = 10 \end{array}$$

If $M >> 0$ very large positive number then $a_2 > 0$ and $a_3 > 0$ makes $z$ to be extremely costly so, the optimal solution force $a_2 = a_3 = 0$.

Note that the R.H.S. of each constraint is non-negative and this is a requirement to start the simplex method.

**At the end of your solution, in other words in the Optimal Tableau two cases are possible:**

(a) If in the Optimal Tableau all artificial variables are equal to 0, the solution is optimal for the given LPP.

(b) If at least one artificial variable is positive in the optimal solution, the LPP is infeasible (no feasible region for our starting LPP).

**Big M Preliminary Form**:

$$\begin{aligned} Row0: &\quad z - 2x_1 - 3x_2 - Ma_2 - Ma_3 = 0 \\ Row1: &\quad 0.5x_1 + 0.25x_2 + s_1 = 4 \\ Row2: &\quad x_1 + 3x_2 - e_2 + a_2 = 31 \\ Row3: &\quad x_1 + x_2 + a_3 = 10 \end{aligned}$$

Here is the solution of Bevco modified LPP starting with the Big M Tableau 0:
**Canonical Tableau 0:**

| $z$ | $x_1$ | $\mathbf{x}_2$ | $s_1$ | $e_2$ | $a_2$ | $a_3$ | rhs | BV | r.t.($x_2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | $2M-2$ | $4M-3$ | 0 | $-M$ | 0 | 0 | $41M$ | $z = 41M$ | |
| 0 | $1/2$ | $1/4$ | 1 | 0 | 0 | 0 | 4 | $s_1 = 4$ | 16 |
| 0 | 1 | 3 | 0 | $-1$ | 1 | 0 | 31 | $a_2 = 31$ | $31/3$ |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 10 | $a_3 = 10$ | $10^* \leftarrow x_2$ |

**BFS** : **BV:** $z = 43M, s_1 = 4, a_2 = 31, a_3 = 10$; **NBV** : $x_1 = x_2 = e_2 = 0$

Solving Row 0 with respect to $z$ we obtain

$$z = (-2M+2)x_1 + (-4M+3)x_2 + Me_2 + 43M$$

to conclude first that the canonical Tableau 0 is not optimal (we have minimum LPP) and second that $x_2$ must enter the basis, in other words to become a BV (basic variable).
Now we have to determine the row, where $x_2$ will enter the basis. The r.t. gives that $x_2$ will enter the basis as BV replacing $a_3$ in Row 3.

**Canonical Tableau 1:**

| $z$ | $\mathbf{x}_1$ | $x_2$ | $s_1$ | $e_2$ | $a_2$ | $a_3$ | rhs | BV | r.t. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | $-2M+1$ | 0 | 0 | $-M$ | 0 | $-4M+3$ | $M+30$ | $z = M+30$ | |
| 0 | $1/4$ | 0 | 1 | 0 | 0 | $-1/4$ | $3/2$ | $s_1 = 3/2$ | |
| 0 | $-2$ | 0 | 0 | $-1$ | 1 | $-3$ | 1 | $a_2 = 1$ | |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 10 | $x_2 = 10$ | |

**BFS** : **BV:** $z = M+30, s_1 = 3/2, a_2 = 1, x_2 = 10$; **NBV** : $x_1 = a_3 = e_2 = 0$

Solving Row 0 with respect to $z$ we obtain

$$z = (2M-1)x_1 + (4M-3)x_2 + Me_2 + (M+30)$$

to conclude that the Canonical Tableau 1 is Optimal Canonical Tableau (we have minimum LPP) and in fact the optimal BFS of the Big M LPP is unique. Why?

The above Canonical Tableau 1 is optimal because all NBV in Row 0 are with negative coefficients. Any attempt to enter one of these variable as BV will increase the value of the objective function but we a looking for MIN. In addition the optimal solution of the Big M LPP is unique. The optimal BFS

**opt. BFS** : **BV:** $z = M + 30, s_1 = 3/2, a_2 = 1, x_2 = 10$; **NBV** : $x_1 = a_3 = e_2 = 0$

has an artificial variable $a_2 > 0$.
**Hence, the given (the original) (modified) Bevco LPP is infeasible, or in other words, the feasible region (FR) of the given (the original) LPP is empty.**

Let us repeat: If at least one of the artificial variable is positive in Optimal Big M Tableau (in other words if it is a BV (basic variable) with positive value), then the original ( the given) LPP does not have a feasible solution or, in other words, the FR of the given (the original) LPP is empty, or in other words, the original (the given) LPP is infeasible.

**Conclusion:** The given (modified) Bevco LPP is infeasible.