1.7. Converting Min LPP to Max LPP and Conversely.

Each Min LPP can be converted to a Max LPP and conversely, each Max LPP can be converted to a Min LPP.

Example 7. Consider DA LPP:

$$\begin{array}{l} \min\ z = 50x_1 + 100x_2 \\ \quad \text{s.t.} \\ 7x_1 + 2x_2 \geq 28 \\ 2x_1 + 12x_2 \geq 24 \\ \hline x_1 \geq 0,\ x_2 \geq 0\,. \end{array} \qquad (1)$$

This is a Min LPP with an optimal solution $x_1 = 3.6$, $x_2 = 1.4$ and $z_{\min} = 320$. Then for each feasible solution (x_1, x_2) (a point from the FR) we have

$$z(x_1, x_2) = 50x_1 + 100x_2 \geq (50)(3.6) + (100)(1.4) = 320$$

and from here

$$-50x_1 - 100x_2 \leq (-50)(3.6) + (-100)(1.4) = -320 \quad \text{for each point } (x_1, x_2) \text{ in the FR.}$$

Hence, the maximum value of the objective function

$$w = -50x_1 - 100x_2$$

on the same as DA LPP feasible region is -320 and this maximum is achieved at the the same as DA LPP optimal solution $x_1 = 3.6$, $x_2 = 1.4$. Thus, the LPP

$$\begin{array}{l} \max\ w = -50x_1 - 100x_2 \\ \quad \text{s.t.} \\ 7x_1 + 2x_2 \geq 28 \\ 2x_1 + 12x_2 \geq 24 \\ \hline x_1 \geq 0,\ x_2 \geq 0\,. \end{array} \qquad (2)$$

has the same as the LPP (1) FR and the same optimal solutions. The optimal $w_{\max}$ value of (2) is connected with the optimal $z_{\min}$ value of (1) by

$$z_{\min} = -w_{\max}.$$

Then, if we are given to solve the Min LPP (1) we can solve the Max LPP (2). The feasible regions of both LPPs coincide, both LPP have the same set of optimal solutions and once we have found $w_{\max}$ we compute z_{min} by the simple formula: $z_{\min} = -w_{\max}$.

Example 8. Both Giapetto's LPP and its corresponding Min LPP have the same feasible region and the same optimal solution:

$$\begin{array}{ll}
\max \ z = 3x_1 + 2x_2 & \min \ w = -3x_1 - 2x_2 \\
\quad \text{s.t.} & \text{s.t.} \\
2x_1 + x_2 \leq 100 & 2x_1 + x_2 \leq 100 \\
x_1 + x_2 \leq 80 & x_1 + x_2 \leq 80 \\
x_1 \leq 40 & x_1 \leq 40 \\
x_1 \geq 0, \ x_2 \geq 0 & x_1 \geq 0, \ x_2 \geq 0\,.
\end{array}$$

In addition, $z_{\max} = -w_{\min}$.