MCG 4345 ASSIGNMENT #1

2.3.1

GIVEN PRESSURE FIELD: $P = x^2 y + y^2$

FIND $\frac{\partial P}{\partial s}$ WHERE s IS AT AN ANGLE OF 45° TO THE x-AXIS

FIND $\frac{\partial P}{\partial s}$ AT (3,2)

$\frac{dx}{ds} = \cos 45°$

$\frac{\partial y}{\partial s} = \sin 45°$

$$\frac{dP}{ds} = \frac{\partial P}{\partial x}\frac{dx}{ds} + \frac{\partial P}{\partial y}\frac{dy}{\partial s}$$

$$\frac{dP}{ds} = (2xy)\frac{dx}{\partial s} + (x^2 + 2y)\frac{\partial y}{\partial s}$$

$$= 2xy(\cos 45°) + (x^2 + 2y)\sin 45°$$

AT (3,2):

$$= 2(3)(2)\frac{1}{\sqrt{2}} + (3^2 + 2(2))\frac{1}{\sqrt{2}}$$

$$= \frac{25}{\sqrt{2}}$$

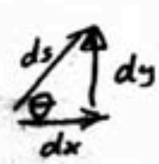
2.3.2

GIVEN PRESSURE FIELD: $P = x^2 y + y^2$

FIND $\frac{dP}{ds}$ AT (3,2) IN THE DIRECTION OF THE CURVE: $3y^2 - 4x = 0$

$y^2 = \frac{4}{3}x$

$$\tan\theta = \frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{\frac{4}{3}x}\right) = \frac{1}{\sqrt{3x}}$$



$\tan\theta = \frac{dy}{dx}$

$\frac{\partial x}{\partial s} = \cos\theta$

$\frac{\partial y}{\partial s} = \sin\theta$

AT (3,2):

$\tan\theta = \frac{1}{3}$, $\theta = 18.43°$

$$\frac{dP}{ds} = \frac{\partial P}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial P}{\partial y}\frac{dy}{ds}$$

$$= (2xy)\cos 18.43° + (x^2 + 2y)\sin 18.43°$$

$$= (2(3)2)\cos 18.43° + (3^2 + 2(2))\sin 18.43°$$

$$= 15.5$$

(CONTINUED)

WHAT IS THE MAXIMUM GRADIENT?

$$\left|\frac{d\rho}{ds}\right|_{max} = |\nabla \rho| = \sqrt{\left(\frac{\partial \rho}{\partial x}\right)^2 + \left(\frac{\partial \rho}{\partial y}\right)^2 + \left(\frac{\partial \rho}{\partial z}\right)^2} \quad \left(\frac{\partial \rho}{\partial z} = 0\right)$$

$$= \sqrt{(2xy)^2 + (x^2+2y)^2}$$

$$\text{AT } (3,2) \quad = \underline{17.7}$$

WHAT IS THE DIRECTION OF THE MAXIMUM GRADIENT?

$$\nabla \rho = \frac{\partial \rho}{\partial x}\hat{i} + \frac{\partial \rho}{\partial y}\hat{j} + \frac{\partial \rho}{\partial z}\hat{k} \quad \left(\frac{\partial \rho}{\partial z} = 0\right)$$

$$= (2xy)\hat{i} + (x^2+2y)\hat{j}$$

ANGLE OF VECTOR ⇒ $\tan\theta = \frac{x^2+2y}{2xy}$

AT $(3,2)$ $\tan\theta = \frac{3^2+2(2)}{2(3)2}$

$$\underline{\theta = 47.3^\circ}$$

CAN ALSO FIND THE DIRECTION OF THE MAXIMUM FROM THE ISOLINES, THE DIRECTION WILL BE NORMAL TO THE ISOLINES.

$\tan\theta = \frac{dy}{dx}$ WHERE ISO SURFACE IS GIVEN BY $x^2y + y^2 = \text{CONST}$

NOTE THAT $\frac{dy}{dx} = \frac{\partial Q}{\partial x}\frac{\partial y}{\partial Q} = \frac{2xy}{x^2+2y}$

AT $(3,2)$: $\tan\theta = \frac{12}{13}$, $\theta = 42.7^\circ$

NORMAL TO THE ISO SURFACE OCCURS WHEN $\theta' = 90 - \theta$

$$\underline{\theta' = 47.3^\circ}$$

2.4.1

GIVEN $u = x^2 y$, $v = -xy^2$

FIND EQUATION OF THE STREAMLINE THROUGH (1,2)

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{x^2 y} = \frac{dy}{-xy^2}$$

$$-\int \frac{1}{x}\,dx = \int \frac{1}{y}\,dy$$

$$-\ln(x) = \ln y + \text{CONST}$$

$$\ln(y) + \ln(x) = \text{CONST}$$

$$\ln(xy) = \text{CONST}$$

$$e^{\ln(xy)} = e^{\text{CONST}}$$

$$xy = \text{CONST}$$

AT (1,2) CONST = 2

$$\boxed{xy = 2}$$