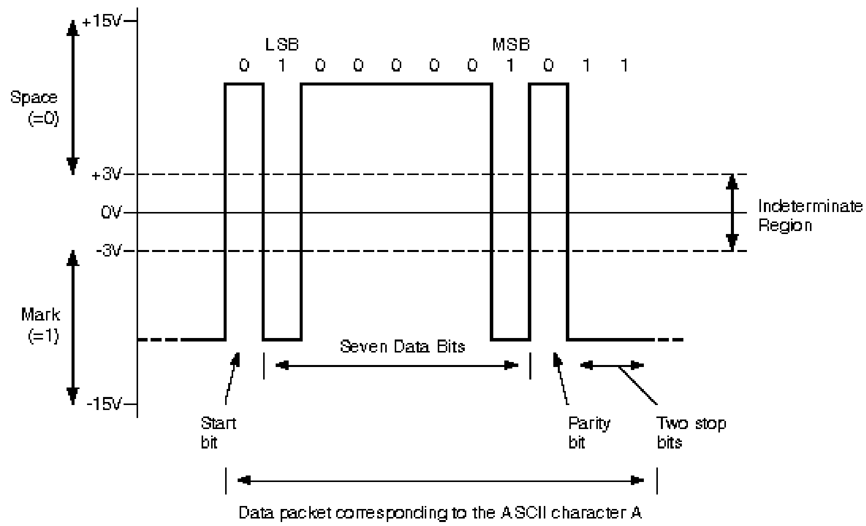
Solutions to Assignment 2

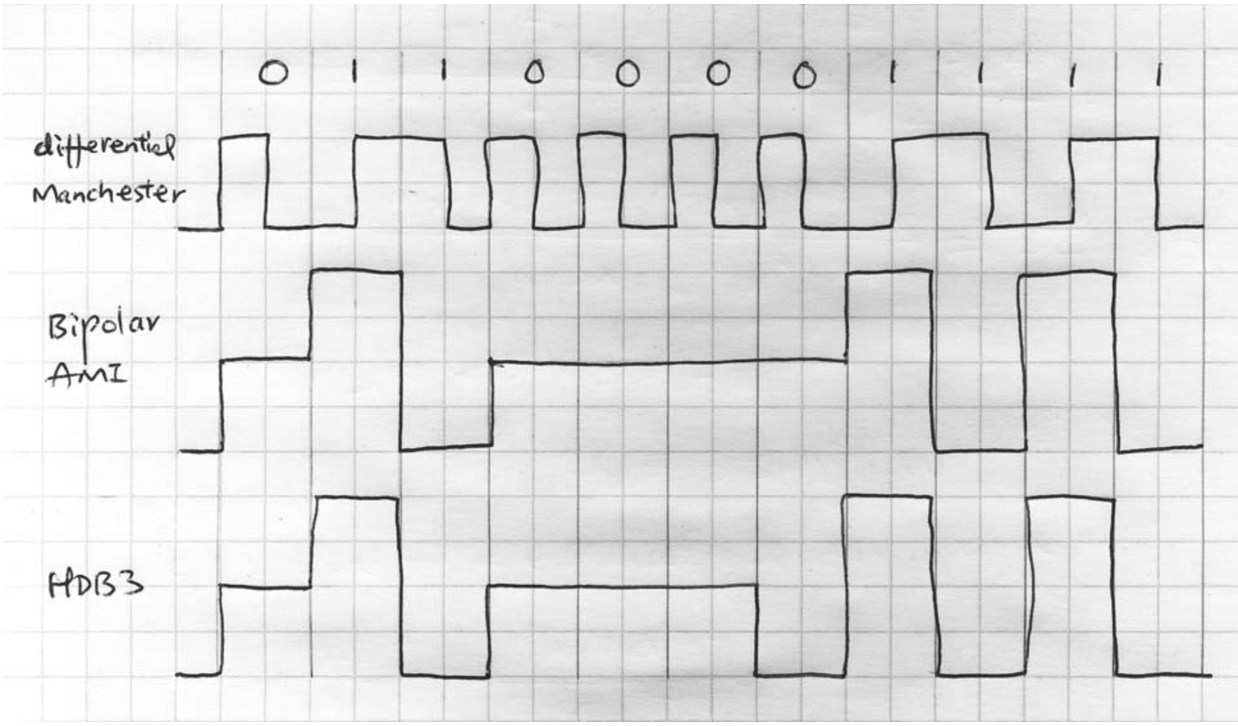
1. [**10 marks**] What encoding scheme is used in the following figure?



In the above, 7-bit ASCII character "A" is encoded as "`01000001011`". What would be the resulting 11 bits for 7-bit ASCII character "a"? Use the same encoding scheme to represent the 11 bits for "a".

Treat the 11 bits for "a" as a sequence of pure data bits (no special consideration for start and stop bits). Use Differential Manchester, Bipolar AMI, and HDB3 schemes to represent the 11 bits for "a". If needed, proper assumptions can be made.

The ASCII code for "A" is 41H, and the ASCII code for "a" is 61H. From the figure, we can see that the even parity is used. The data packet corresponding to the ASCII character "a" is 01100001111.



2. [**10 marks**] Consider an audio signal with spectral components in the range 300 to 3000 Hz. Assume that a sampling rate of 7000 sample per second will be used to generate a PCM signal.
   a. Is the sampling rate high enough?
   b. For quantization noise SNR_{dB}=45 dB, what is the number of uniform quantization needed?
   c. What data rate is required?
   d. If the dynamic range of the signal is -5V to +5V, what are the PCM codes for voltage levels -4.9V and +3V?

a) The sampling rate is sufficient according to Nyquist theorem. The minimum required sampling rate is 3000*2=6000 Hz.
b) According to the equation, $SQNR_{dB}=10\log_{10}2^n+1.76dB=6.02n+1.76dB$, when SNR_{dB}=45 dB, n is 8 bits, which gives 256 quantization levels.
c) The data rate then is 8 bits/sample * 7000 samples/second, which is 56000 bps.
d) -4.9V: (-4.9-(-5))/(5-(-5))*255=3 → 00000011.
   +3V: (3-(-5))/(5-(-5))*255=204 → 11001100.

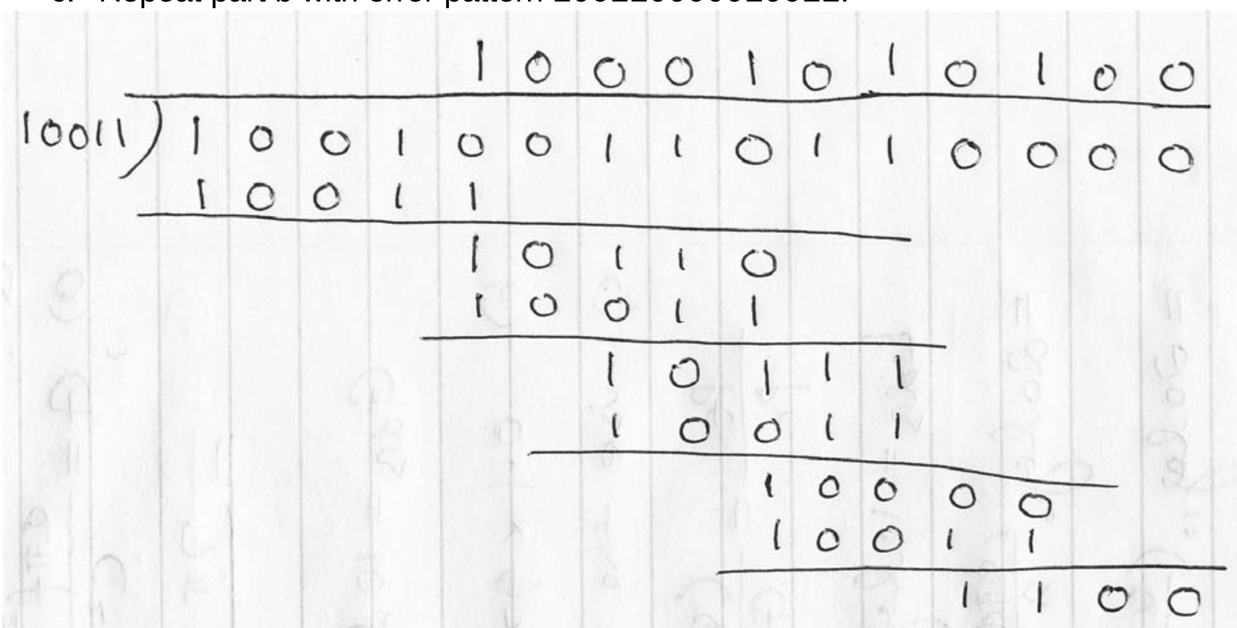
3. [**10 marks**] Consider an asynchronous transmission system that transfer N data bits between a start bit and a stop bit. What is maximum value of N if the receiver clock is 1% faster than the transmitter clock?

Assume the date rate is x bps. The interval (bit length) for each bit is 1/x second.
The sender and receiver sample at the middle of each bit.
If the receiver's clock is faster, the receiver's sample points will gradually shift to left.
As long as the receiver's sample point is in the correct interval, there will be no error.

If the receiver’s clock is 1% faster, for each bit, the left shift is 1/x-1/(1.01x).

Consider the start bit, $\frac{\frac{1}{2x}}{\frac{1}{x}-\frac{1}{1.01x}}=1+N$. N=49.

4. [**10 marks**] In a CRC error-detecting scheme, the generator polynomial is $X^4 + X + 1$.
   a. Encode the bits 10010011011.
   b. Suppose the channel introduces an error pattern 100010000000000 (i.e., a flip from 1 to 0 or 0 to 1 in position 1 and 5). Can the error be detected?
   c. Repeat part b with error pattern 100110000010011.



a). The encoded bits are 100100110111100.
b). This pattern can be detected because it is not the multiple of 10011.
c). This pattern cannot be detected because it is the multiple of 10011.

5. [**10 marks**] Explain clearly why a CRC-16 or CRC-CCITT can detect all burst errors of length 16 or less, 99.997% of 17-bit error bursts, and 99.998% of 18-bit and longer bursts.

[excerpted from Tanenbaum’s textbook]

A polynomial code with r check bits will detect all burst errors of length <= r. A burst error of length k can be represented by $x^i(x^{k-1}+\cdots+1)$, where i determines how far from the right-hand end of the received frame the burst is located. If the generator polynomial, G(x), contains an x^0 term, it will not have x^i as a factor, so if the degree of the parenthesized expression is less than the degree of G(x), the reminder can never be zero.

If the burst error length is r+1, the reminder of the division by G(x) will be zero if and only if the burst is identical to G(x). By definition of a burst, the first and last bits must be 1, so whether it matches depends on the r-1 intermediate bits. If all combinations are regarded as equally likely, the probability of such an incorrect frame being accepted as valid is $1/2^{r-1}$, which is 99.997% for 16-bit CRC.

When an error burst longer than r+1 bits occurs, the probability of a bad frame getting through unnoticed is $1/2^{r}$, which is 99.998% for 16-bit CRC.

6. **[10 marks]** A channel has a data rate of *R* bps and propagation delay of *t* s/km. The distance between the sending and receiving nodes is *L* kilometers. Nodes exchanges fixed-size frames of *B* bits. Find a formula that gives the minimum sequence field size of the frame as a function of *R*, *t*, *B*, and *L* (considering maximum utilization). Assume that ACK frames are negligible in size and the processing at the nodes is instantaneous.

This is issue of sliding window. If the window size is W, then the sender can send W frames without having to wait for ACK from the receiver. It takes time for a frame to travel from sender to receiver and also it takes the same amount of time for an ACK frame to travel from the receiver to the sender. The return trip time is $t = 2\times L\times t$. The time needed for sending out a frame is $t_{frame} = B/R$. The number of frames that can be transmitted during the time period t is $N = \left\lceil \frac{t}{t_{frame}} + 0.5 \right\rceil = \left\lceil \frac{2\times L\times t}{B/R} + 0.5 \right\rceil$. The number of bits needed for representing N is $M = \lceil \log_2 N \rceil$.

7. **[10 marks]** This question is about sliding window. For a k-bit sequence number field, which provides a sequence number range from 0 to 2^k-1, the maximum window size is limited to 2^k-1. Use k=3 as an example to explain why the window size cannot be 8.

Let's consider the following situation. If k=3 and window size=8, the sequence number of frames can range from 0 to 7. Suppose there are frames 0 through 7 in the sliding window of the receiver:

01234567[01234567]0123...

The sender sent out frames 0 through 7 and is waiting for the ACK for frame 0. The receiver received these 8 frames successfully and sent back an ACK 0. Then the sliding window of the receiver became:

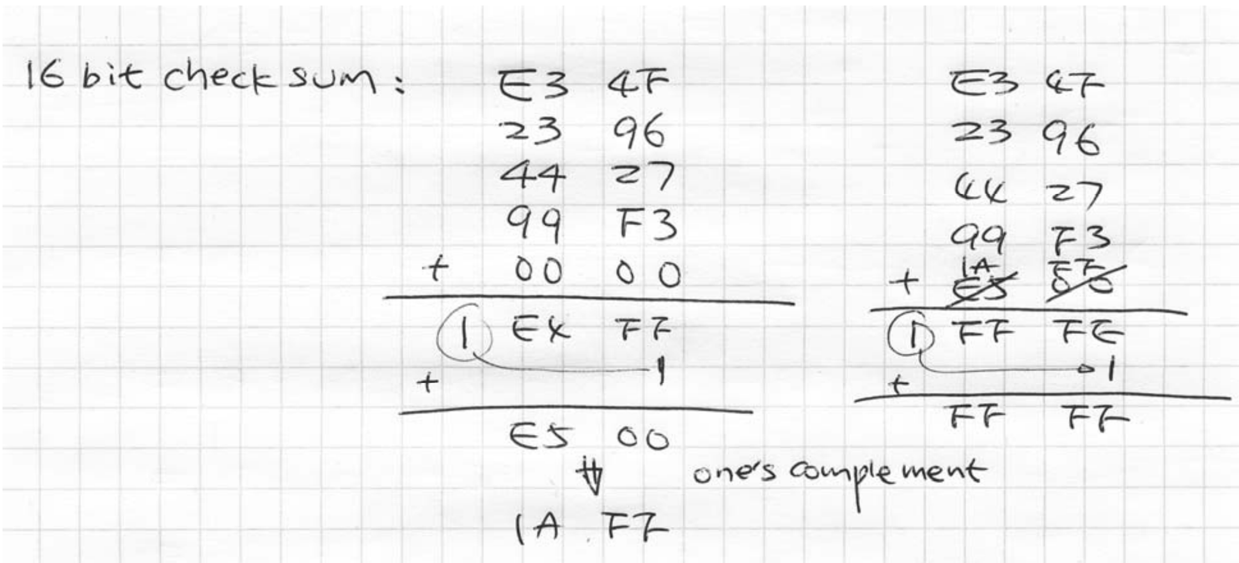
0123456701234567[01234567]0123...

The ACK is lost due to some reasons. The sender didn't receive the expected ACK before the time-out timer expires. The sender re-sends the frames starting from 0. The receiver receives the re-sent frames. It cannot distinguish whether these are expected frames or repeated frames, so frame duplication happens.

```
If we limit the sliding window size to 7 the problem will not happen (see
below). The receiver is expecting frame 7. If it receives frame 0, it
knows for sure that the frame 0 is a repeated frame which needs to be
ignored.

012345670123456701234...
0123456701234567012345670123...
```

8. **[10 marks]** Compute the checksum for the data block E3 4F 23 96 44 27 99 F3. Then perform the verification calculation.



9. [**20 marks**] You are experimenting with the design of a digital intercom system. Using twisted pair wire, you have connected two such intercoms between your house and garage via a pair of modems. An engineer friend has measured the SNR of the twisted pair line to be 35 dB and available bandwidth of 10 kHz (note that the spectrum must be divided into two bands to allow the simultaneous (full-duplex) communications between the two intercoms).

   Initially, you have set the intercom parameters to 50 dB SNR for quantization noise and PAM sampling rate of 8000 samples per second to sample the voice spectrum of 0-4 kHz.

   The modems are capable of the baud rates 1200, 2400, 4800, and 9000 baud and can be configured with modulation schemes QAM-16, QAM-32, QAM-64, and QAM-128.

Can you make the intercoms work? In particular, specify the intercom parameters to deliver a suitable voice signal (must deliver signal spectrum 300 to 3500 Hz, with minimum 35 dB quantization SNR) and the modem settings (baud rate and modulation scheme). Provide an explanation of your configuration.

The Shannon capacity is

$SNR_{dB}=10\log_{10}(SNR)$, $SNR=10^{3.5}=3162$
The bandwidth is 10kHz/2=5kHz.
$C = B \log_2 (1+SNR) = 5000 \log_2(1+3162) = 58135$ bps

The modem can have the following configurations:

	1200 baud	2400 baud	4800 baud	9000 baud
QAM-16 (4 bits)	4,800 bps	9,600 bps	19,200 bps	36,000 bps
QAM-32 (5 bits)	6,000 bps	12,000 bps	24,000 bps	45,000 bps
QAM-64 (6 bits)	7,200 bps	14,400 bps	28,800 bps	54,000 bps
QAM-128 (7 bits)	8,400 bps	16,800 bps	33,600 bps	63,000 bps

Considering the Shannon capacity, the highest modem speed we can use is 54000 bps, which is QAM-64 with 9000 bauds.

In order to support 50dB signal-to-quantization-noise ratio, number of bits per sample should be (50-1.76)/6.02= 9 bits. With a sampling rate of 8000, we generate 9*8000=72 kbps. This rate is not supported by either the Shannon capacity or Modem. So we have to make compromise, i.e., to lower the sampling rate or the number of bits per sample.

The minimum requirement is “must deliver signal spectrum 300 to 3500 Hz, with minimum 35dB signal to quantization noise ratio”. The highest frequency is 3500 Hz. According to the Nyquist theorem, the minimum sampling rate should be 7000 Hz. Also, (35-1.76)/6.02=6 bits. That means, as long as the sampling rate is higher than 7000 Hz, and number of bits per sample is higher or equal to 6, we meet the minimum requirement.

To meet the minimum requirement, a bit rate of 6*7000=42000 bps is needed. 54000 bps (QAM-64 with 9000 bauds) can support the requirement. To use the full capacity of modem, we can choose to use 8 bits/sample and 7000 sample/second to five 8*7000=56000 bps.