

Lecture 2 SCM

-Obj of SCM: get product to market efficiently (less costly) to increase firm's effectiveness in meeting consumer demand

- drivers: ongoing to improve ops & efficiency | increased outsourcing | free trade & globalization | increasing e-commerce | control inventories

Bullwhip Effect

-def: increase in demand variability as move up SC

-causes: isolation of mfg from retail | errors in forecasting | promotional pricing

-implication: strain on mfg capacity resulting in costly overtime | rescheduling | increasing inventory holdings—increase costs & shelf life issues

-remedies: make product to order & ↑inventories

-problems of making product to order? → havoc in scheduling; ↓plant efficiencies (excess expenses); ↑direct costs; disrupt delivery & distri schedules, ↑delivery costs; ↓customer service (unanticipated shortfalls in finished goods inventories that can occur when previously planned product is interrupted)

-stock up & ship out of larger finished product inventories? → problems with stale & outdated product; ↑costs of warehouse space; ↑carrying costs

-risk maintaining too small inventory → short shipments; not being able to maintain shelf space risk; customer dissatisfaction & LT customer loss

-solution → balance between lot sizes; advge of econ of scale & small enough to avoid excess inven

-what to do with date-sensitive products? → distribution time (shipment & distri) & time on store shelves before purchase and consumption

Supply Chain Activities

-operational → forecasting, purchasing, scheduling, warehouse & inventory control, customer service functions

-operational decisions → 1) inventory ↑ in values as progressing thru production process; 2) product become more specific as move down product line

-factor for inventory decisions:

*risk pooling: holding safety stocks in one location NOT multiple

*delayed differentiation/postponement

Hierarchy of decisions in OM

Strategic Decisions (Long-Range)

- What, how, where to operate

- Products/services, processes, facilities

Tactical Decisions (Intermediate)

- How much, when

- Operations planning, inv. planning

Operational Decisions (Short-Range)

- Plan & control ops. to meet demand

- Operations scheduling, worker assignment, quality control

Logistics

-def: movement & storage of materials, finished goods & info occurring both within facility & outside of facility

*incoming goods, finished products, miscellany supplies, fuel & liquid, maintenance parts, handling returning goods

-logistical costs: within the plant to do the above duties; 3rd party costs (jobbing some responsibilities to others); handling inventory outside of company = financing + spoilage + obsolescence + shrink

-traffic mgmt.: selection, methods, scheduling (in & out)

-selecting mode of transportation → assume: cost is determining, buyer takes ownership at shipment & responsible for freight costs, competing transportation are equally reliable

In-transit holding cost = $H(d)/365$

Total delivery costs

= freight (transport) costs + in-transit holding costs

*H is annual holding costs of goods shipped, d is # of days in transit

-Just-in-Time delivery: (QR) quick response, (ECR) efficient consumer response, (VMI) vendor managed inventory — PULL

*impact: ↑ traffic volume, ↑ in transport costs/unit, risk shortage due to unanticipated delivery disruptions

-distribution requirements planning: built upon forecasting estimates & derived by working backwards thru SC to determine time-phased replenishment rates of getting goods from mfg system thru to each level of distri—PUSH

eCommerce

-global in nature—service, consumers

-↑competitiveness & accessibility

-consumer demand info collection

-product customization

-↓ response time, costs, transaction costs

-virtual companies, level the playing field

Effective SC requirement

-form close relationship based on trust, coop, agreement on common goals

-effective comm channels to facilitate improved coordination of SC activities

-SC visibility & info sharing

-event mgmt. —SC members respond to items

-performance measurement

5 Steps Create an effective SC

-develop strategic obj & tactics → integrate & coordinate internal activities → coordinate with suppliers & customers → coordinate planning & exec across SC → form strategic partnership

Collaborative planning, forecasting & replenishment

-test sincerity & effectiveness of co-op & effective SC

3 steps: 1) plan on specifics of product focus, timeline promo, order lead time, discounts, data sharing; 2) initial forecasting (by buyer, supplier assess production capacity, agreement); 3) replenishment (buyer places order & supplier delivers)

Performance metrics

-Quality: costs of good sold (invent turnovers, unit costs, warehouse & logistics costs); flexibility; delivery (lead time, # of on-time deliveries); customer service (fill rate, response time from order entry to shipment)

-inventory turnover, fill rate

PURCHASING

-def: procurement of RM, parts needed for mfg, repair parts to existing machine, general office, plant supplies, acquisition of required services

-importance: cost control, quality, timing of deliveries, LT industry partnerships

-interfaces: legal (contract, disputes), accounting (payment), data process (inventory record), design (specs, info on new stuff), receiving (info on shipment), suppliers (vendor analysis), ops(main sources of request)

Purchasing cycle

Receive requisition (descript of items, quan, qual, deliv) → select supplier → place order → monitor → receive order → pay supplier

-def: assess function of particular mfg to see if it can be modified or replaced to ↓ costs without sacrificing functionality

-criteria&process: articulate function of input → ask question: substitute materials, combo of parts, easing of functional specific, pkg alter

Value analysis

Collect, cleasing, classifying, & analyzing expenditure data with purpose of ↓ procurement costs, ↑ efficiency, monitor compliance

-agility: respond to ST changes in demand or supply quickly—cha: info sharing, collab, keep inv avoid delay

-adaptability: adjust SC design to accommodate market changes —cha: know econ, flex thru commonality

-alignment: establish incentives for SC partners to improve performance of entire chain —challenge: info sharing, share risk, align incentives

Spend analysis

Collect, cleasing, classifying, & analyzing expenditure data with purpose of ↓ procurement costs, ↑ efficiency, monitor compliance

Triple-A SC

-agility: respond to ST changes in demand or supply quickly—cha: info sharing, collab, keep inv avoid delay

-adaptability: adjust SC design to accommodate market changes —cha: know econ, flex thru commonality

-alignment: establish incentives for SC partners to improve performance of entire chain —challenge: info sharing, share risk, align incentives

Types of SC

-efficient SC (highest cost efficiency) - achieve econ of scale by eliminating waste & optimizing techniqu

-risk-hedging (pooling & sharing resources) - eliminate supply disruption by pooling & sharing resources

-responsive (responsive & flex) - utilize strategies to become more responsive & flex to the ever-changing needs & wants of customer

-agile (hedged & responsive) - combine strengths of other 3 SC

Outsourcing

-benefits: lower costs (econ of scale), gain expertise & knowledge, flex, ↑ capacity

-risk: reduction in control & expertise

Supply Chain Win-Win Contracts

EXAMPLE: walmart order cost \$80/unit, sell \$125/unit, salvage value \$20

Underestimating $Q < D$, revenue = price * Q; Cost = cost * Q

Overestimating $Q > D$, revenue = price * D + salvage * (Q - D)

Cost = cost * D

-Ce = cost of excess = 80 - 20; Cs = cost of shortage = 125 - 80

Prob of demand $\Rightarrow P(D < Q) = Cs / (Cs + Ce)$

= 45 / (60 + 45) = 0.43 then find the higher cumulative value than critical ratio — $Q = 12000$

Expected profit = $\$125 * 12000 - 80 * 12000 = \540000 with prob of 0.28

Ce = purchase \$ - salvage value = c - s; Cs = selling \$ - purchase \$ = p - c

Decision Rule

Order $Q + 1$ if $F(Q) < Cs / (Cs + Ce)$; do not order $Q + 1$ if $F(Q) > Cs / (Cs + Ce)$; indifferent if equal

SC Contract

EXAMPLE: FC = \$100000, VC = \$35, C = wholesale price = \$80, selling price = \$125, salvage value = \$20

Buyback (new salvage value)

Buyback price = \$55 → cost of shortage = 125 - 80 = 45, cost of excess = 80 - 55 = 25 → cost of excess goes down

Q < D: retailer → revenue = S * Q, cost = wholesale S * Q; manufacturer → revenue = wholesale S * Q, cost = FC + VC * Q

Q > D: retailer → revenue = price * D + buyback * (Q - D), cost = wholesale S * Q; manufacturer → revenue = wholesale S * Q + salvage * (Q - D), cost = FC + VC * Q + buyback * (Q - D)

Revenue Sharing

Revenue shared = 15%, wholesale price = \$60 → revenue earned by retailer = 125(1 - 0.15) = 106.25, R = portion by mfg = 125 - 106.25 → cost of excess = \$60 - 20 = \$40

Q < D: retailer → revenue = price * Q, cost = wholesale S * Q + R * price * Q; manufacturer → revenue = wholesale S * Q + R * price * Q, cost = FC + VC * Q

Q > D: retailer → revenue = price * D + salvage * (Q - D), cost = wholesale S * Q + R * price * D; manufacturer → revenue = wholesale S * Q + R * price * D, cost = FC + VC * Q

Global optimization: integrated SC

NO wholesale price → cost of shortage = 125 - 35 = 90, cost of excess = 35 - 20 = 15

Optimum production quantity? Cs = 125 - 35 = 90, Ce = 35 - 20 = 15; $P(D < Q) = Cs / (Cs + Ce) = 90 / 105 = 0.86$ → set $Q = 16000$

Q < D → revenue = price * Q, cost = FC + VC * Q

Q > D → revenue = price * D + salvage value * (Q - D), cost = FC + VC * Q

Chapter 12: Inventory Mgmt

Invent: meet anticipated dem & prod econ of scale (cycle invent), avoid stock-outs (safety invent), prod/delivery takes time (pipeline inv.), maintain indep of prod. — distri ops (decoupling inv./buffer), confront demand fluctuations (seasonal inv)

IMgmt: keep track of inv., reliable forecast, lead time knowledge/variability, estimates costs, classification

EOQ: strike balance b/w the costs associated with holding inv. & the costs incurred in ordering prod.

-ordering freq. will ↓ inv cost by ↓ avg on-hand inv.

-infreq. Will ↓ order process cost, ↑ inv. Holding cost

Quality discount: price discount used as an incentives for purchasers to order in large Q

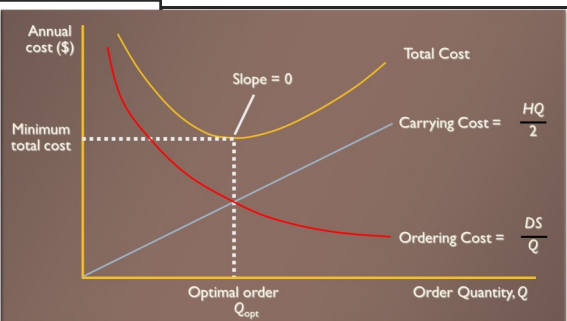
Obj: to compare benefits of purchase price savings for buying in Q as opposed to the ↑ in annual holding costs of maintaining large avg inv.

All unit: price/unit is the same for all units ordered

Incremental: price/unit ↓ in stages for diff. portions

Relevant cost: acquisition cost (\$/item purchased); carrying cost = holding cost (\$/item/time out); ordering costs (\$/order); shortage costs (\$/item short)

-carry cost = holding cost: cost of capital, insurance, shrinkage, material handling



QO minimizes TC (Q)

Determine ROP with EOQ

EXAMPLE: lead time = 5 days, avg daily demand = 10.12 units, avg demand during lead time = 5 * 10.12 = 51 units

ROP = daily demand * lead time = 51 units → reorder when it drops to 51 units

Optimal # of orders = D/Q
Time between orders = $1/\text{# of orders} = Q/D$

EOQ example 1

Demand constant rate = 2000 units/year, \$0.5 to replenish stock, \$0.2 to carry item in stock, receive stock 4 days after order, 300 working days/year

$Q_0 = \sqrt{\frac{2 \times 2000 \times 0.50}{0.20}}$ $Q_0 = \sqrt{\frac{2 \times 10000 \times 75}{6}}$

$Q_0 = 100$ units $Q_0 = 500$ units

$ROP = \frac{2000}{300} \times 4 = 75 = \frac{10000}{500} + 6 \times \frac{500}{2}$

$ROP = 26.7 = 27$ units $= \$3000$ per year

EOQ example 2

Demand 10000, H = \$6/unit, order cost = \$75

Production instead of purchasing?

-involve setup, produced in batches for efficiency

-goal: determine proper batch size/lot size

-large batch size: less setups but more inv cost → optimal size: Econ Production Quantity

Cycle length: $T = Q_0 / D$

Uptime: $T_1 = Q_0 / P$; Downtime: $T_2 = T - T_1$

Maximum inventory: $Q_0 \times (1 - D/P)$

Relevant cost = setup cost + carrying cost

TC = $S \times N + H' \times I_{avg}$

TC(Q) = $S \times D/Q + H' \times Q/2$

Economic Production Quantity (EPQ)

$Q_0 = \sqrt{\frac{2 \times D \times S}{H \times (1 - D/P)}}$

Holding cost = $H' = h(1 - D/P)$ → P = production rate

Batch size Q0 determines: prod run time, prod interval, inventory, freq. of setups

EPQ example

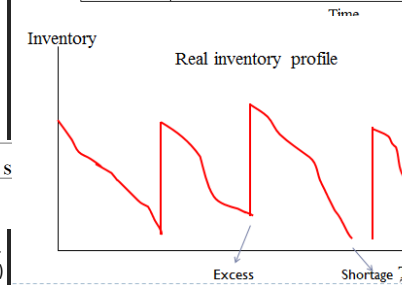
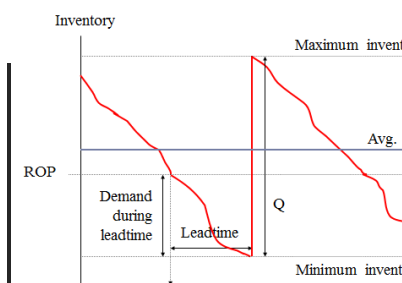
Demand = 20000 bags/day, 5 days/week, 50 weeks/yr, prod = 50000/day, setup = \$200, annual H = \$0.55/bag

$H = \$0.55$ per bag per year $Q_0 = \sqrt{\frac{2 \times D \times S}{H \times (1 - D/P)}}$

$S = \$200$

Demand is 20,000 bags per day $D = 20,000$ bags × 50 wks × 5 days

Production is 50,000 bags per day



Total cost = acquisition cost + order cost + carrying cost + shortage cost

Relevant cost = cost which depends on order size & freq. = order cost + carrying cost

Econ order Q derivation (purchasing)

S = order cost, Q = order quantity, H = carrying cost, N = number of orders/year, D = demand, Iavg = avg inv.

$TC = S \times N + H' \times I_{avg} = S \times D/Q + H \times Q/2$

$$Q_0 = \sqrt{\frac{2 \times D \times S}{H}}$$

$TC(Q_0) = S \times \frac{D}{Q_0} + H' \times \frac{Q_0}{2}$, where $H' = H(1 - D/P)$

= $200 \times \frac{500000}{77850} + 0.55(1 - 5mil/12.5mil) \times \frac{77850}{2}$

= \$25690 per year

Cycle length = $Q_0/D = 77850/20000$, uptime = $T_1 = Q_0/P = 77850/50000$

ABC Classification (cumulative %)

A: 5-15% of units, 70-80% of value

B: 30% of units, 15% of value (more attention)

C: 50-60% of units, 5-10% of value (easy system)

Transportation Mode Selection

A company in Toronto receives motors from a supplier in Winnipeg. Demand for motors is 120,000 units per year. The cost of each motor is \$120.

Train:
 - Charges \$2 per motor, takes 7 days
 - Safety stock needed in Toronto is 3,000 units
 Holding cost (both in-transit and safety) is 25% of total unit cost per year

Truck:
 - Charges \$4 per motor, takes 3 days
 - Safety stock needed in Toronto is 1,000 units

Based on total cost (annual freight, in-transit holding, and safety stock holding), which mode of transportation should be used?

Solution

Mode	Annual freight
truck	120,000(\$4) = \$480,000
train	120,000(\$2) = \$240,000
Mode	Annual in-transit holding cost
truck	120,000(\$120)(.25)(3/365) = \$29,589.04
train	120,000(\$120)(.25)(7/365) = \$69,041.10
Mode	Annual Safety Stock holding cost
truck	1,000(\$120 + \$4)(.25) = \$31,000
train	3,000(\$120 + \$2)(.25) = \$91,500
Mode	Annual Total cost
truck	\$540,589.04
train	\$400,541.10 ← cheaper

A phone company has annual demand of 10,000 units. The cost of each phone is \$100. However, quantity discounts apply as follows.

Units	Discount (%)
1-49	-
50-99	2.5
100-149	5.0
150-199	7.5
200-249	12.0
250+	15.0

The holding cost is 25% per unit per year. The ordering cost is \$75. Assume 250 working days a year. What is the best order quantity to use?
 D = 10,000 units per year. S = \$75. H = .25P

Quantity	Cost	Discount (%)	Holding Cost	EOQ
1-49	\$ 100.00	-	\$ 25.00	
50-99	\$ 97.50	2.5	\$ 24.38	
100-149	\$ 95.00	5.0	\$ 23.75	
150-199	\$ 92.50	7.5	\$ 23.13	
200-249	\$ 88.00	12.0	\$ 22.00	
250+	\$ 85.00	15.0	\$ 21.25	265.68

A smartphone manufacturer is currently producing the circuit boards that they use in their new line of phones. The forecasts suggest an annual demand of 30,000 for the brand new smartphone. The setup cost of production is \$50 and they can produce with a rate of 120,000 circuit boards a year. The unit cost of production is estimated as \$40 per board and the annual inventory carrying cost as 20% of unit value.

a) What is the optimal run size and what percentage of the time will the production be running?

$$H = 0.2 \times 40 = \$8$$

$$H' = 8 \left(1 - \frac{30,000}{120,000} \right) = 8 \times 0.75 = \$6$$

$$EPQ = \frac{\sqrt{(2 \times 30,000 \times 50)}}{6} = 708$$

Percentage of the time machine is running = uptime / production cycle = $\frac{T_1}{T}$
 $= \frac{30,000}{120,000} = 0.25 = 25\%$

b) What is the minimum total annual cost for carrying and setup?

$$TC(EPQ) = \sqrt{2DSH'} = \sqrt{2 \times 30,000 \times 50 \times 6} = \$4242.6$$

c) Recently, it was brought to the attention of the VP-Operations that there is an offer from an outside supplier to provide the circuit boards for the company for \$38 per board. In that case, the ordering cost will be \$400 per order. The total cost is the only decision-making factor that the VP-Operations is considering at the moment. Should she make the boards in-house or outsource them to the outside supplier, and what is the cost associated with the decision?

Purchasing:
 Holding cost for the purchasing option = $0.2 \times 38 = \$7.6$
 Total carrying and setup cost for the purchasing = $\sqrt{2 \times 30,000 \times 50 \times 7.6} = \$13,505.55$
 Total cost = acquisition cost + TC(EOQ) = $30,000 \times 38 + 13,505.55 = \$1,153,505.55$

Production:
 Total cost associated with the production option = production cost + optimal cost of setup and inventory carrying cost = $30,000 \times 40 + 4242.6 = \$1,204,242.6$

• purchasing-production • outsource

Example: Demand=D=50 units, H=\$200/unit/year, B=Back-order cost per unit per year=\$500, ordering cost=S=\$10/order. Quantity=? Back-order quantity/order cycle=?

$$Q = \sqrt{\frac{2DS}{H} \left(\frac{H+B}{B} \right)} = \sqrt{\frac{2(50)(10)}{200} \left(\frac{200+500}{B} \right)} = 1.65$$

$$Q_b = Q \left(\frac{H}{H+B} \right) = 3 \left(\frac{200}{200+500} \right) = 0.86, \text{ round to } 1$$

Allow inventory to drop to zero, when another unit is needed, order 3 units

EOQ: determine how many to order, minimize TC by balancing carrying cost & order cost, robust & work even if all assumptions are not met

Difference between EOQ & EPQ?

-EPQ is mainly for producing batches, EOQ is for receiving orders, quantity is received gradually for EPQ

Example ROP

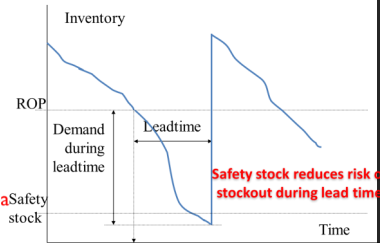
Annual demand = 1,000 units; Days per year = 365; Lead time = 7 days

$$d = \frac{1,000 \text{ units/year}}{365 \text{ days/year}} = 2.74 \text{ units/day}$$

$$ROP = d * L = 2.74 \text{ units/day} * 7 \text{ days} = 19.18 \text{ or } 20 \text{ units}$$

→ when inv. reach 20 units, place next order

Inventory Control with uncertainty in D & LT



ROP=Expected demand + safety stock during LT

Safety stock: stock that is held in excess of expected demand due to variability of D and/or lead time

1) variability of D & LT, 2) service level (probability demand will not exceed supply)

→ LT service level & annual service level

Determinants of ROP

-rate of D, LT, D & or LT variability, stockout risk(SS)

EOQ with quantity discounts

EXAMPLE: • \$0.10 per copy for 1-9 copies; \$0.08 per copy for 10-49 copies

• 9 units ordered, total = \$0.10 * 9 = \$0.90; 10 units total = \$0.08 * 10 = \$0.80 → So, order 10 units; throw away the 10th unit and save \$0.10!

Relevant cost=acquisition cost + order cost + carrying cost = R*D+S*N+H*I*avg = R*D+S*D/Q+H*Q/2

EXAMPLE:

Range	P	H
1-199	\$1	0.17*I=\$0.17
200-499	\$0.94	\$0.1598
500+	\$0.87	\$0.1479

EOQ(500+)=sq root(2*400*22/0.15)=343 → not feasi.

EOQ(200-499)=sq root(2*400*22/0.16)=332 → feasi.

TC(Q)=HQ/2 + SD/Q + R*I; TC(332)=\$429.07; TC(500)=\$403.1

EXAMPLE

Nick's Camera Shop carries Zodiac disposable cameras

The cameras normally costs Nick \$3.20 per unit, and he sells them for \$5.25 per unit

Nick's average sale is 21 cameras per week (52 weeks in a year)

His annual inventory holding cost rate is 25% and it costs Nick \$20 to place an order with Zodiac

If Zodiac offers a 7% discount on orders of 400 units or more and a 10% discount for 900 units or more, determine Nick's optimal order quantity

$$EOQ_{2.88} = \sqrt{2 * 1092 * 20 / 0.72} = 246.3 = 247$$

247 < 900 not feasible, simply use 900 as order quantity

► D = 21 per week

► D = 21 units / week × 52 weeks / year = 1,092 per year

► Annual holding cost rate is 25%

Range	Unit Price R	Holding cost H
1 - 399	\$3.20	.25(3.20) = \$0.80
400 - 899	\$2.98	.25(2.98) = \$0.75
900+	\$2.88	.25(2.88) = \$0.72

EOQ_{2.88} = 900 units

$$EOQ_{2.98} = \sqrt{2 * 1092 * 20 / 0.75} = 241.3 = 242$$

242 < 400 not feasible, simply use 400 as order quantity

EOQ_{2.98} = 400 units

$$EOQ_{3.20} = \sqrt{2 * 1092 * 20 / 0.80} = 233.7 = 234$$

234 < 399 feasible, use 234

EOQ_{3.20} = 234 units

EOQ with planned shortage

-large holding cost/unit, demand can wait, company

$$TC(Q_{break-point(2.88)}) = 2.88 \times 1092 + 20 \times 1092 / 900 + 0.72 \times 900 / 2 = \$3,493.23 \text{ per year}$$

$$TC(Q_{break-point(2.98)}) = 2.98 \times 1092 + 20 \times 1092 / 400 + 0.75 \times 400 / 2 = \$3,453.19 \text{ per year}$$

$$TC(EOQ_{3.20}) = 3.20 \times 1092 + 20 \times 1092 / 234 + 0.8 \times 234 / 2 = \$3,681.33 \text{ per year}$$

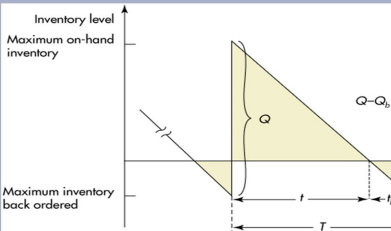
may intentionally allow shortage

assume: short D is back-ordered, back-order with Cs, Cs is proportional to waiting time

B = back-order cost per unit per year

Q_b = Quantity back-ordered per order cycle

T = Order interval



Symbol	Description
D	Consumer Demand
Q _o	Order Quantity
Q _s	Quantity Sold=D
S	Retail Price
C	Retail Cost
P	Var. Prod. Cost
F	Fix Prod. Cost
V	Salvage Value
Q _E	Excess Inv.
B	Buyback Value

Scenario	Decision Maker	Excess Inv. Resp. Q _E = Q _O - D	Retailer [I]	Manufacturer [II]
Sequential	Retailer	Retailer	$\Sigma [(Q_s * S + Q_E * V) * \text{prob}] - Q_o * C$	$Q_o(C - P) - F$
Global	Manufacturer	Manufacturer	N/A	$\Sigma [(Q_s * S + Q_E * V) * \text{prob}] - (Q_o * P) - F$
Buyback	Retailer	Shared	$\Sigma [(Q_s * S + Q_E * B) * \text{prob}] - Q_o * C$	$Q_o(C - P) - F - \Sigma [(B - V) * \text{prob}]$
Revenue Sharing	Retailer	Retailer	$\Sigma [[(1 - R) * Q_s * S + Q_E * V] * \text{prob}] - Q_o * C$	$Q_o * (C - P) + \Sigma [(R * Q_s * S) * \text{prob}] - F$

Solution Process – use Short Cut Method to determine optimal quantity – use Table Method to determine profitability in conjunction with the formulas listed above – compare results of sequential scenario to global scenario to determine potential benefits – consider use of Buyback or Revenue Sharing Contracts to increase welfare of both retailer and manufacturer

Description	Basic EOQ	EOQ (Discounts)	EPQ
Average Inventory		Q/2	I _{max} /2
Annual Holding Cost		(Q/2)H	(I _{max} /2)H
Number of Orders		D/Q	D/Q
Annual Ordering (Setup) Costs		(D/Q)S _o	(D/Q)S _s
Efficient Quantity		sqrt(2DS _o /H)	sqrt(2pDS _s /H(p-q))
Maximum Inventory Level		-	(Q/p)(p-d)
Length of Order Cycle (days)		365(Q/D)	Q/d
Total Inventory Cost Function		(Q/2)H + (D/Q)S _o	(I _{max} /2)H + (D/Q)S _s
Production Length		-	Q/p

$$TC = \left(\frac{Q - Q_b}{2Q} \right) H + \left(\frac{D}{Q} \right) S + \left(\frac{Q_b}{2Q} \right) B$$

$$Q = \sqrt{\frac{2DS}{H} \left(\frac{H+B}{B} \right)}; Q_b = Q \left(\frac{H}{H+B} \right)$$

B = back - order cost per unit per year

Q_b = quantity back - ordered per order cycle

H = annual holding cost per unit

D = annual demand

S = ordering (or setup) cost per order

$$t = \frac{Q - Q_b}{d}, t_b = \frac{Q_b}{d}$$

$$I_{avg} = \frac{(Q - Q_b)^2}{2Q}$$

$$Q_b_{avg} = \frac{(Q_b)^2}{2Q}$$