

**Name:** \_\_\_\_\_**Section:** \_\_\_\_\_**Instructor:** \_\_\_\_\_

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**Instructions:**

- (1) You must show supporting work to receive credits.
- (2) Use exact values whenever possible, e.g.  $\pi/4$  instead of 0.785398...
- (3) Calculators are allowed.
- (4) Make sure you have all the 7 pages of questions.

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Point	14	16	16	16	16	18	14	18	12	16	14	14	16	200
Credit														

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1. (14 points) Let  $f(x, y) = \sqrt{4x + 5y}$ .
  - (a) (10 points) Find the quadratic approximation to  $f(x, y)$  at the point  $(4, -3)$ .

- (b) (4 points) Use the quadratic approximation of (a) to approximate  $f(4.1, -2.9)$ .

2. (16 points) Let  $S$  be the surface given by  $\frac{6 - z^2}{x^2 + y^2} = 1$ .

(a) (4 points) Verify that point  $P = (1, 1, 2)$  is on the surface.

(b) (6 points) Find a normal vector of  $S$  at the point  $P$ .

(c) (6 points) Write down an equation of the tangent plane to  $S$  at  $P$ .

3. (16 points) Use polar coordinates to evaluate the following integral:

$$\int_0^2 \int_0^{\sqrt{4-y^2}} e^{x^2+y^2} dx dy.$$

4. (16 points) Let  $f(x, y) = 3xy^2 + 12xy - 4x^2$ .

(a) (8 points) Find all critical points of  $f$ .

(b) (8 points) Classify each critical point of  $f$  as a local maximum, a local minimum or a saddle point.

5. (16 points) Use the method of Lagrange Multipliers to find the point on the plane  $4x - 3y + 5z = 48$  closest to the point  $(0, 4, -3)$ . (*Hint*: Minimize the square of the distance from  $(0, 4, -3)$  to  $(x, y, z)$ .)

6. (18 points) Let  $f(x, y, z) = 2x^2 \ln(y) + xe^{2z}$ , let  $P = (-2, 1, 0)$  and  $Q = (2, 5, -2)$ .

(a) (6 points) Find and simplify  $\nabla f(-2, 1, 0)$ .

(b) (6 points) Find and simplify the rate of change of  $f$  at  $P$  in the direction from  $P$  to  $Q$ .

(c) (6 points) Find and simplify the unit vector in the direction in which  $f$  decreases most rapidly at  $P$ .

7. (14 points) Consider  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x+z) dz dy dx$ . Change the integral to an iterated triple integral in the spherical coordinates. (**DO NOT EVALUATE the triple integral.**)

8. (18 points) Let  $\vec{F}(x, y) = (x^2 + 2xy)\vec{i} + (x^2 + 5y + 1)\vec{j}$ .

(a) (6 points) Use a derivative test to verify that the vector field  $\vec{F}(x, y)$  is a conservative vector field.

(b) (8 points) Find a potential function  $f$  for  $\vec{F}(x, y)$ .

(c) (4 points) Find the  $\int_C \vec{F}(x, y) \cdot d\vec{r}$ , where  $C$  is a smooth path from  $(1, 0)$  to  $(0, 1)$ .

9. (12 points) Consider the integral  $\int_0^2 \int_{x^4}^{8x} f(x, y) dy dx$ .

(a) (4 points) Sketch or describe the region of integration.

(b) (8 points) Switch the order of integration.

10. (16 points) Let  $\vec{F}(x, y) = -y\vec{i} + x\vec{j}$  and  $C$  be the line from  $(1, 0)$  to  $(2, 3)$ .

(a) (6 points) Find a parameterization of  $C$ .

(b) (10 points) Find the line integral  $\int_C \vec{F} \cdot d\vec{r}$ .

11. (14 points) Let  $S$  be the portion of the paraboloid  $z = 4 - x^2 - y^2$  lying above the  $xy$ -plane, with upward orientation. Find the flux  $\int_S \vec{F} \cdot d\vec{A}$  through  $S$ , where the vector field is  $\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$ .

12. (14 points) Suppose  $\vec{F}$  is a smooth vector field defined everywhere such that  $\operatorname{div}\vec{F} = 10$ . Find the flux of  $\vec{F}$  out of a closed cylinder (with cover and base) of height 5 and radius 2, centered on the  $z$ -axis with base in the  $xy$ -plane.

13. (16 points) Let  $\vec{F} = (y + 2z)\vec{i} + 4x\vec{j} + yz\vec{k}$ .

(a) (6 points) Find the curl of  $\vec{F}$ :  $\nabla \times \vec{F}$ .

- (b) (10 points) Use your result from part (a) to find the line integral around the circle of radius 1 in the  $xy$ -plane, centered at the origin, oriented counterclockwise when viewed from above.