Solution Devoir 5

Question 1

(a) $\mathcal{F}\{3\cos 2\pi(490)t + \cos 2\pi(510)t\}$

$$= \frac{3}{4}\delta(f-490) + \frac{3}{4}\delta(f-510) + \frac{3}{4}\delta(f+490) + \frac{3}{4}\delta(f+510)$$

(b) $\mathcal{F}\{12\cos 2\pi(1000)t + \cos 2\pi(1000)t\}$

$$= 3\delta(f-900) + 3\delta(f-1100) + 3\delta(f+900) + 3\delta(f+1100)$$

$\mathcal{F}\{12\sin 2\pi(1000)t + 5\sin 2\pi(1000)t\}$

$= ?$

$$\mathcal{F}\{12\sin 2\pi(1000)t\} = \frac{6}{j}\delta(f-1000) - \frac{6}{j}\delta(f+1000)$$

$$\mathcal{F}\{m(t)\sin 2\pi(1000)t\} = \frac{1}{2j}M(f-1000) - \frac{1}{2j}M(f+1000)$$

$\therefore \mathcal{F}\{12\sin 2\pi(1000)t + 5\sin 2\pi(1000)t\}$

$$= -3\delta(f-1100) + 3\delta(f-900) - 3\delta(f+1100) + 3\delta(f+900)$$

thus $\mathcal{F}\{12\cos 2\pi(100)t\cos 2\pi(1000)t + 12\sin 2\pi(100)t\sin 2\pi(1000)t\}$

$= 6\delta(f-900) + 6\delta(f+900)$

(Lower sideband AM)

(c) $\Lambda(5t)\cos 2\pi 300t$

$\mathcal{F}\{\Lambda(5t)\} = \frac{1}{5}\text{sinc}^2\left(\frac{f}{5}\right)$

$\mathcal{F}\{\Lambda(5t)\cos 2\pi 300t\}$

$= \frac{1}{10}\text{sinc}^2\left(\frac{f-300}{5}\right)$

$+ \frac{1}{10}\text{sinc}^2\left(\frac{f+300}{5}\right)$

Question 2

$y(t) = \frac{dx(t)}{dt}$

$Y(f) = j2\pi f X(f)$

$Y_h(f) = -j\,\text{sgn}(f)\,Y(f)$

$= -j\,\text{sgn}(f)\,j2\pi f X(f)$

$= j2\pi f\left(-j\,\text{sgn}(f)X(f)\right)$

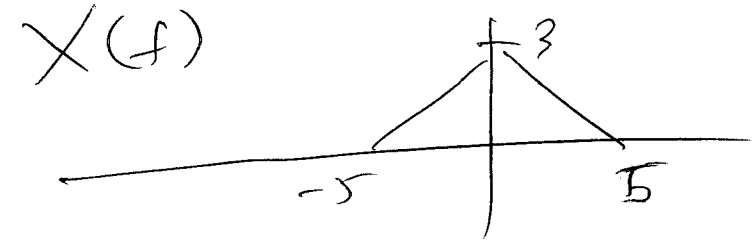
$= j2\pi f X_h(f)$

alors

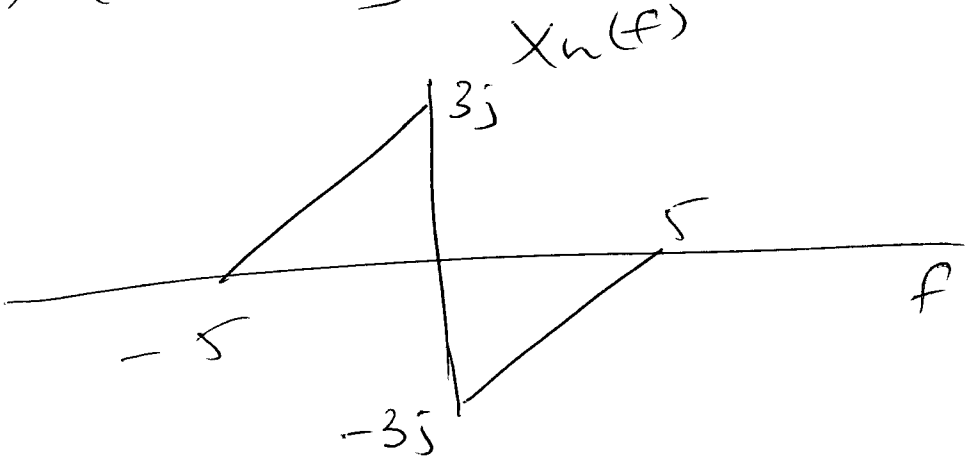
$$y_n(t) = \mathcal{F}^{-1}\{j2\pi f X_n(f)\}$$

$$= \frac{d\, x_n(t)}{dt}$$

Question 3



$$X_n(f) = -j\,\mathrm{sgn}(f)\,X(f)$$



(That's all there was to this question. C'est tout !)

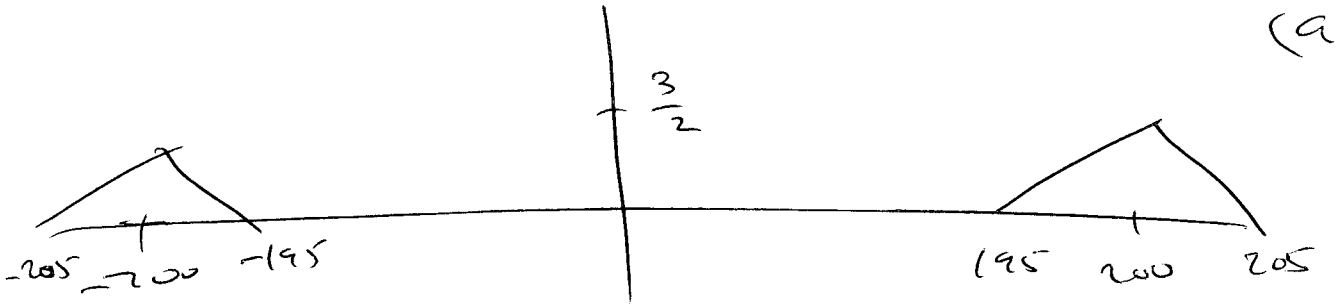
Question 4

$$\mathcal{F}\{x(t)\cos 2\pi 200t + x_h(t)\sin 2\pi 200t\}$$

$$= \frac{1}{2}X(f-200) + \frac{1}{2}X(f+200) \quad \rightarrow (a)$$

$$+ \frac{1}{2j}X_h(f-200) - \frac{1}{2j}X_h(f+200) \quad \rightarrow (b)$$

(a)



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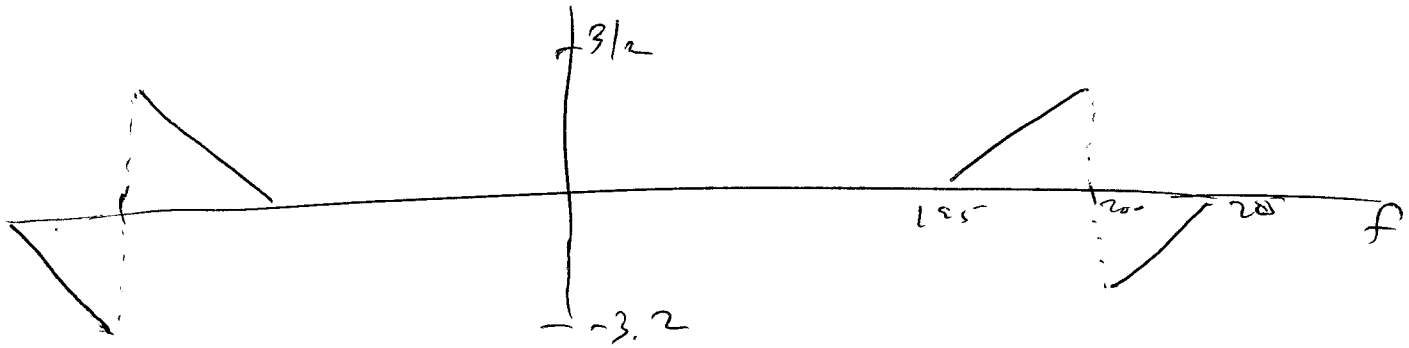

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