COMM 220 – PRACTICE PROBLEM SET 2 SOLUTIONS

1. (a)

K	L	Q
1	1	9
2	2	13.2475
4	4	19.5595
8	8	28.9706

When the inputs are doubled, output increases by less than double. This production function exhibits decreasing returns to scale.

(b) When K=8, $Q = f(8,L) = 6L^{1/2} + 3(8)^{2/3} = 6L^{1/2} + 12$

$AP_L = Q/L = 6/L^{1/2} + 12/L$

(c) $MP_L = dQ/dL = 0.5(6)L^{-1/2} = 3/L^{1/2}$

2. (a) $MP_L = dQ/dL = 20 - 0.1L$

$AP_L = Q/L = -80/L + 20 - 0.05L$

(b) AP_L reaches its maximum when $AP_L = MP_L$

$-80/L + 20 - 0.05L = 20 - 0.1L$

$L = (80/0.05)^{0.5} = 40$, where AP_L reaches its maximum

At $L = 40$, $MP_L = 20 - 0.1(40) = 16$

3. (a) $MRTS_{LK} = MP_L/MP_K = 250(0.5)K^{0.75}L^{-0.5} / 250(0.75)K^{-0.25}L^{0.5} = 2K/3L$

(b)

K	L	Q
1	1	250
2	2	594.6036
4	4	1414.2136
8	8	3363.5857

When the inputs are doubled, output increases by more than double. This production function exhibits increasing returns to scale.

(c) $MP_L = 125K^{0.75}/L^{0.5}$. Since L is in the denominator, as L increases, MP_L decreases. For example, for any given value of K:

$L = 1$, $MP_L = 125K^{0.75}$

$L = 2$, $MP_L = 88.3883K^{0.75}$

$L = 3$, $MP_L = 72.1688K^{0.75}$

$MP_K = 187.5L^{0.5}/K^{0.25}$. Since K is in the denominator, as K increases, MP_K decreases. For example, for any given value of L:

$K = 1$, $MP_K = 187.5L^{0.5}$

$K = 2$, $MP_K = 157.6681L^{0.5}$

$K = 3$, $MP_K = 142.4692L^{0.5}$

(d)

K	L	Q	MP_L
100	1	7905.6942	-
100	2	11180.3399	3274.6457
100	3	13693.0639	2512.7240
100	4	15811.3883	2118.3244

As shown in the table above, MP_L is decreasing as L increases.