Solutions Test 1 MATH1104I

1. $\begin{pmatrix} 3 & 1 & -2 & 5 \\ 2 & 0 & -6 & -4 \\ 2 & 2 & 1 & 4 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 2 & 0 & -6 & -4 \\ 3 & 1 & -2 & 5 \\ 2 & 2 & 1 & 4 \end{pmatrix} \xrightarrow{R_1 = \frac{1}{2}R_1} \begin{pmatrix} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 2 & 2 & 1 & 4 \end{pmatrix}$

$\xrightarrow[R_3 = R_3 - 2R_1]{R_2 = R_2 - 3R_1} \begin{pmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 2 & 7 & 8 \end{pmatrix} \xrightarrow{R_3 = R_3 - 2R_2} \begin{pmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & -7 & -14 \end{pmatrix}$

$$\begin{aligned} x_1 - 3x_3 &= -2 \\ x_2 + 7x_3 &= 11 \\ -7x_3 &= -14 \end{aligned}$$

$$\begin{aligned} x_3 &= 2 \\ x_2 + 14 &= 11 \\ x_2 &= -3 \\ x_1 - 6 &= -2 \\ x_1 &= 4 \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}.$$

2. $\begin{pmatrix} 0 & 1 & -2 & 1 & 1 \\ 2 & -1 & 0 & -1 & 0 \\ 4 & 1 & -6 & 1 & 3 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 2 & -1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 & 1 \\ 4 & 1 & -6 & 1 & 3 \end{pmatrix}$

$\xrightarrow{R_3 = R_3 - 2R_1} \begin{pmatrix} 2 & -1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 3 & -6 & 3 & 3 \end{pmatrix} \xrightarrow{R_3 = R_3 - 3R_2} \begin{pmatrix} 2 & -1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$\xrightarrow{R_1 = R_1 + R_2} \begin{pmatrix} 2 & 0 & -2 & 0 & 1 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 = \frac{1}{2}R_1} \begin{pmatrix} 1 & 0 & -1 & 0 & \frac{1}{2} \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_3 + \frac{1}{2} \\ 2x_3 - x_4 + 1 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

3: $BC^T = \begin{bmatrix} -2 & 1 & -2 \\ 1 & 0 & -2 \\ -1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 0 & -3 \\ 1 & -3 & -4 \\ -3 & 0 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 2 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 2 \\ -2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -4 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$-2A^2 = \begin{bmatrix} 2 & -2 & 0 \\ 8 & -6 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

$BC^T - 2A^2 = \begin{bmatrix} -1 & -2 & -3 \\ 9 & -9 & -4 \\ -3 & 0 & -2 \end{bmatrix}$

4: a) $\begin{bmatrix} 3 & -2 \\ -4 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$Ax = b$

b) $A^{-1} = \frac{1}{12-8}\begin{bmatrix} 4 & 2 \\ 4 & 3 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 4 & 2 \\ 4 & 3 \end{bmatrix}$

$x = A^{-1}b = \frac{1}{4}\begin{bmatrix} 4 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ 5/2 \end{bmatrix}$

5) $\left[\begin{array}{ccc|ccc} 1 & -6 & -4 & 1 & 0 & 0 \\ 2 & -10 & -9 & 0 & 1 & 0 \\ -1 & 6 & 5 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_2 = R_2 - 2R_1 \\ \longrightarrow \\ R_3 = R_3 + R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & -6 & -4 & 1 & 0 & 0 \\ 0 & 2 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$

$R_1 = R_1 + 3R_2 \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -7 & -5 & 3 & 0 \\ 0 & 2 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 = R_1 - 7R_3 \\ \longrightarrow \\ R_2 = R_2 + R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 3 & 7 \\ 0 & 2 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$

$R_2 = \frac{1}{2}R_2 \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 3 & 7 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \quad \therefore I^{-1} = \left[\begin{array}{ccc} 2 & 3 & 7 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 1 \end{array}\right]$

6) $\begin{pmatrix} 1 & h & 2 \\ 4 & 8 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & h & 2 \\ 0 & 8-4h & -5 \end{pmatrix}$

So we need

$8 - 4h \neq 0$

$8 \neq 4h$

$\boxed{h \neq 2}$

7) $(A^T B^{-1} C)^{-1} = C^{-1}(B^{-1})^{-1}(A^T)^{-1}$

$= \boxed{C^{-1} B (A^{-1})^T}$