

**FACULTY OF ENGINEERING AND COMPUTER SCIENCES  
CONCORDIA UNIVERSITY  
APPLIED ADVANCED CALCULUS (ENGR 233) FINAL EXAMINATION  
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**Special instructions: Do all problems  
No calculators are allowed**

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**PROBLEM No. 1 (10 MARKS)**

- (a) Find the equation of the plane which contains the three points (2, 1, 0), (3, 4, 0), and (1, 1, 1)
- (b) Find the parametric equations of the line through (3, 4, 0), which is perpendicular to the plane found in part (a).

**PROBLEM No. 2 (10 MARKS)**

- (a) Let  $w = \sqrt{x^3 + y} + e^{xz}$ , and let  $x = 2t$ ,  $y = t^2$ , and  $z = t^{-1}$ . Use the multivariate chain rule to find  $\frac{dw}{dt}$  when  $t = 1$ .
- (b) Suppose that the temperature distribution of the plane is given by  $T(x,y) = 5x^2 + e^{xy}$ . If an ant is sitting at the point (2,3), in which unit direction should it move in order to cool off as fast as possible? What is the directional derivative of  $T$  in that direction?

**PROBLEM No. 3 (10 MARKS).** Find point(s) on the surface  $z = 10 - x^2 - y^2$  at which the tangent plane is parallel to the plane  $x + \frac{3}{2}y + \frac{1}{2}z + d = 0$ , where  $d$  is a constant.

**PROBLEM No. 4 (10 MARKS).** Find the moment of inertia about the  $y$ -axis of the lamina that has the given shape and density.

$$\text{bounded by } x = 0, y = x, y = 1; \rho(x, y) = \sqrt{1 + y^4}$$

**PROBLEM No. 5 (10 MARKS).** For the scalar function

$$f(x, y, z) = e^{x^2} \cos z + z^4 \sin y$$

compute the following quantities if they **make sense**. If not, explain why.

- (a) **grad f**   (b) **div(grad f)**   (c) **grad(div f)**   (d) **curl(grad f)**   (e) **grad(grad f)**

**PROBLEM No. 6 (10 MARKS).** Compute the line integral

$$\int_C -y \, dx + x \, dy$$

for the following curves

- (a)  $C$  is the curve from  $(3,0)$  to  $(-3,0)$  lying along the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  in the  $xy$  plane,  $y \geq 0$ .
- (b)  $C$  is the closed curve in the  $xy$  plane described, in the counterclockwise sense, by  $r = 2 \cos \theta$ ,  $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ .

**PROBLEM No. 7 (10 MARKS).** Prove that the line integral

$$I = \int_C (1 + e^{-y}) \, dx - (xe^{-y} + 4y) \, dy$$

is independent of the path, and then evaluate  $I$  when  $C$  is any path from  $(1, 0)$  to  $(2, 1)$ .

**PROBLEM No. 8 (10 MARKS).** Find the outward flux of the radial vector field

$\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  through the boundary of domain in  $\mathbb{R}^3$  given by two inequalities  $x^2 + y^2 + z^2 \leq 2$  and  $z \geq x^2 + y^2$ .

**PROBLEM No. 9 (10 MARKS).**  $D$  is the region of three-dimensional space that is given by the inequalities  $x^2 + y^2 + z^2 \leq 1$ ,  $4z^2 \leq x^2 + y^2 + z^2$  and  $z \geq 0$ . Find the volume of  $D$ .

**PROBLEM No. 10 (10 MARKS).** A field of force:  $\mathbf{F} = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$  exists within a region of space.

- (a) Solve  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where the closed contour “ $C$ ” is the intersection of  $x^2 + y^2 = 1$  and  $z = y^2$  using Stokes Theorem. Show all of your work and justify your answer.
- (b) If the object were moved along the closed path “ $C$ ” 16 times, how much more work would be done than if the object were moved along the closed path just once? Justify your answer.